UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BEng (Hons) AUTOMOTIVE PERFORMANCE ENGINEERING

SEMESTER 1: EXAMINATION

ENGINEERING MATHEMATICS II

MODULE NUMBER: MSP5017

Date: 17^{th} **January 2019** **Time: 2.00pm – 4.00pm**

INSTRUCTIONS TO CANDIDATES:

1. **Answer all SIX questions.**
2. **The examination paper carries a maximum of 100 marks.**
3. **Maximum marks for parts of each question are shown in brackets.**
4. **This examination is open book.**

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1. Consider the following equation:

$$e^{-3x} + 2x^2 = 6$$

   (a) Show there is a solution to the equation on the interval $[1, 5]$ of the real line. (2 marks)

   (b) Use the method of bisection *once* to determine an initial approximate solution to the equation. (3 marks)

   (c) Using the approximation in (b) as your initial value, use the Newton-Raphson method to find a solution to the equation accurate to 3 significant figures. (10 marks)

2. Consider the following differential equation for $x(t)$:

$$(\star) \qquad \begin{cases} \dot{x} + 4x = 10e^{-6t} \\ x(0) = 4 \end{cases}$$

   where $\dot{x}$ denotes first-order differentiation of x with respect to t.

   (a) Show the Laplace transform of $(\star)$ yields:

$$X(s) = \frac{4s + 34}{s^2 + 10s + 24}$$

   where $X(s) = \mathcal{L}\{x(t)\}$ is the Laplace transform of the function $x(t)$. (8 marks)

   (b) Obtain the solution to $(\star)$ by taking a partial fraction decomposition of $X(s)$ and using inverse Laplace transforms. (14 marks)

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3. Consider the following second-order ordinary differential equation for the function $x(t)$:

$$\begin{cases} \ddot{x} - 3\dot{x} - 4x = 1 \\ x(0) = 0 \\ \dot{x}(0) = 1 \end{cases}$$

where $\dot{x}$ and $\ddot{x}$ denote first and second-order differentiation of x with respect to t respectively.

(a) Show that the second-order differential equation can be written as the *system of first-order ordinary differential equations*:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = 1 + 3y_2 + 4y_1 \\ y_1(0) = 0 \\ y_2(0) = 1 \end{cases}$$

in terms of two suitably chosen new variables $y_1(t), y_2(t)$. (6 marks)

(b) Use the method of Laplace transforms to show the system of ordinary differential equations can be written as the system of *algebraic equations*:

$$\begin{aligned} sY_1(s) - Y_2(s) &= 0 \\ -4Y_1(s) + (s-3)Y_2(s) &= \frac{s+1}{s} \end{aligned}$$

where $Y_1(s) = \mathcal{L}\{\, y_1(t)\, \}$ and $Y_2(s) = \mathcal{L}\{\, y_2(t)\, \}$. (6 marks)

(c) Solve the system of algebraic equations to show that $Y_1(s) = \dfrac{1}{s(s-4)}$. (6 marks)

(d) Take the inverse Laplace transform of $Y_1(s)$ to find the solution to the original second-order ordinary differential equation. (6 marks)

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4. Consider the following double integral: $$\mathcal{I} = \int_{x=0}^{3}\left(\int_{y=0}^{6-2x}(2xy + 4e^{y})\,dy\right)dx.$$

   (a) Sketch the region of integration. (4 marks)

   (b) Change the order of integration in $\mathcal{I}$, using your diagram to obtain the new limits of integration. (4 marks)

   (c) Use either expression for the integral to show that $\mathcal{I} = 13 + 2e^{6}$. (10 marks)

5. The wave equation in one spatial dimension is given by:

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$$

   where $u = u(t,x)$ is a function of two variables and v is some positive constant.

   (a) Show, by substitution into the wave equation, the following function is a solution:

$$u(t,x) = \sin(x)\cos(vt) + 10e^{x-vt}$$

   (6 marks)

   (b) One method of solving the wave equation is by *separation of variables*. Explain this method and apply it to the wave equation to reduce the partial differential equation to two *ordinary differential equations*. (8 marks)

6. Show that *conservation of mass* of a fluid in one spatial dimension can be written as the continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

   where t, x denote time and the spatial co-ordinate respectively; $\rho = \rho(t,x)$ is the mass density of the fluid; and $v = v(t,x)$ is the velocity of the fluid. (7 marks)

END OF PAPER