# UNIVERSITY OF BOLTON

# SCHOOL OF ENGINEERING

## BEng (Hons) AUTOMOTIVE PERFORMANCE ENGINEERING

# SEMESTER 1: EXAMINATION

# ENGINEERING MATHEMATICS

# MODULE NUMBER: MSP4017

**Date:** $15^{th}$ **January 2019** **Time: 2.00pm – 4.00pm**

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**INSTRUCTIONS TO CANDIDATES:**

1. **Answer all EIGHT questions.**
2. **The examination paper carries a maximum of 100 marks.**
3. **Maximum marks for parts of each question are shown in brackets.**
4. **This examination is open book.**

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1. Let $f$ denote the quadratic function:

$$f(x) = x^2 + 4x + 8.$$

(a) Show that $x_1 = -2 + 2i$ and $x_2 = -2 - 2i$ are roots of $f(x) = 0$. (4 marks)

(b) Plot the roots $x_1, x_2$ on an Argand diagram. (3 marks)

(c) Simplify $\dfrac{x_2}{x_1}$ and write it in the form $a + bi$ for some real numbers $a$ and $b$. (4 marks)

2. Let $x, y$ be sinusoidal functions defined by

$$x(t) = 5\cos\left(\omega t + \frac{\pi}{3}\right) \quad \text{and} \quad y(t) = 2\cos\left(\omega t + \frac{\pi}{4}\right).$$

Use the method of phasors to combine $x(t) + y(t)$ into a single sinusoidal function. (8 marks)

3. Let $\underline{v}, \underline{w}$ be two vectors defined by

$$\underline{v} = \begin{pmatrix} 4 \\ \lambda \\ 1 \end{pmatrix} \quad \text{and} \quad \underline{w} = \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$$

with respect to the standard basis in Cartesian coordinates on $\mathbb{R}^3$.

(a) Find the value of $\lambda$ for which the vectors $\underline{v}$ and $\underline{w}$ are *perpendicular*. (3 marks)

(b) With $\lambda = 1$, find a *unit vector* in $\mathbb{R}^3$ that is perpendicular to both $\underline{v}$ and $\underline{w}$. (4 marks)

(c) With $\lambda = 2$, find the angle between the vectors $\underline{v}$ and $\underline{w}$. (4 marks)



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4. (a) Let $f$ be the function defined by $f(x) = 3x^2 + 2x - 1$. Find the derivative of $f$ *from first principles*. (4 marks)

   (b) Calculate the first derivative of the functions $f, g, h$ defined by the following:

   (i) $f(x) = x^2 \sin(3x)$ (4 marks)

   (ii) $g(x) = \sqrt{9 - x^2}$ (4 marks)

   (iii) $h(x) = \dfrac{\cos(3x)}{x^2 + 6}$ (4 marks)

5. Calculate the following definite integrals:

   (a) $\displaystyle\int_{x=0}^{\frac{\pi}{4}} 8\cos(2x)\,dx$ (4 marks)

   (b) $\displaystyle\int_{x=0}^{1} 12xe^{-6x}\,dx$ (5 marks)

   (c) $\displaystyle\int_{x=0}^{2} 2xe^{1-2x^2}\,dx$ using the substitution $u = 1 - 2x^2$. (5 marks)

6. (a) Let $(a_k)$ be a sequence defined by $a_k = 3k - 1$.

   (i) Write down the first 5 terms of the sequence $(a_k)$. (2 marks)

   (ii) Calculate $\sum_{k=1}^{20} a_k$. (4 marks)

   (b) Let $f$ be the function defined by $f(x) = e^{-2x}$. Write down the third-order Maclaurin series expansion of $f(x)$. (5 marks)

   (c) Use a generalised binomial expansion to find a second-order rational approximation of $\sqrt{27}$. (5 marks)



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7. Headlight bulbs are made by a machine and the lifespan of 11 such bulbs are recorded:

| Lifetime (hours) | 92 | 64 | 86 | 77 | 91 | 76 | 55 | 66 | 85 | 79 | 87 |
|---|---|---|---|---|---|---|---|---|---|---|---|

(a) Find the mean lifetime of a bulb made by the machine. (2 marks)

(b) Find the standard deviation of the data. (4 marks)

(c) Draw a box-and-whiskers plot displaying the data in the table above. (6 marks)

8. (a) The probability a component is manufactured to an acceptable standard is 0.47. Twelve components are picked at random. Calculate the probability that:

(i) five are acceptable; (3 marks)

(ii) more than ten are acceptable. (3 marks)

(b) A radar unit is used to measure speeds of cars on a motorway. The speeds are normally distributed with a mean of $62$ mph and a standard deviation of $10$ mph. What is the probability that a car selected at random will be travelling:

(i) between $60$ and $70$ mph; (3 marks)

(ii) more than $80$ mph. (3 marks)



**END OF PAPER**