

**UNIVERSITY OF BOLTON**  
**SCHOOL OF ENGINEERING**  
**MSC SYSTEM ENGINEERING**  
**SEMESTER ONE EXAMINATION 2018/2019**  
**SIGNAL PROCESSING**  
**MODULE NO: EEM7011**

Date: Wednesday 16<sup>th</sup> January 2019      Time: 14:00 – 16:00

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**INSTRUCTIONS TO CANDIDATES:**      There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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**Question 1.**

- a) Discuss briefly the conditions necessary for a realisable digital filter to have a linear phase characteristic and the advantage of filters with such characteristics.

**[5 marks]**

- b) An FIR filter has its impulse response,  $h[n]$  defined over interval  $0 \leq n \leq N-1$ . Show that if  $N=8$  and  $h[n]$  satisfies the following symmetry condition:

$$h[n] = h[N-1-n],$$

the phase provided by the filter is linear in nature whose generic value is given by

$$\text{Angle } H(e^{j\omega}) = -\frac{N-1}{2}\omega$$

**[20 marks]**

**Total 25 marks**

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**Question 2.**

- a) Given the Z-transform is given by  $x(z) = \sum_{n=-\infty}^{n=+\infty} x(n)z^{-n}$ , consider the system given by the following equation:

$$x[n] = \left[ \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n \right] u(n)$$

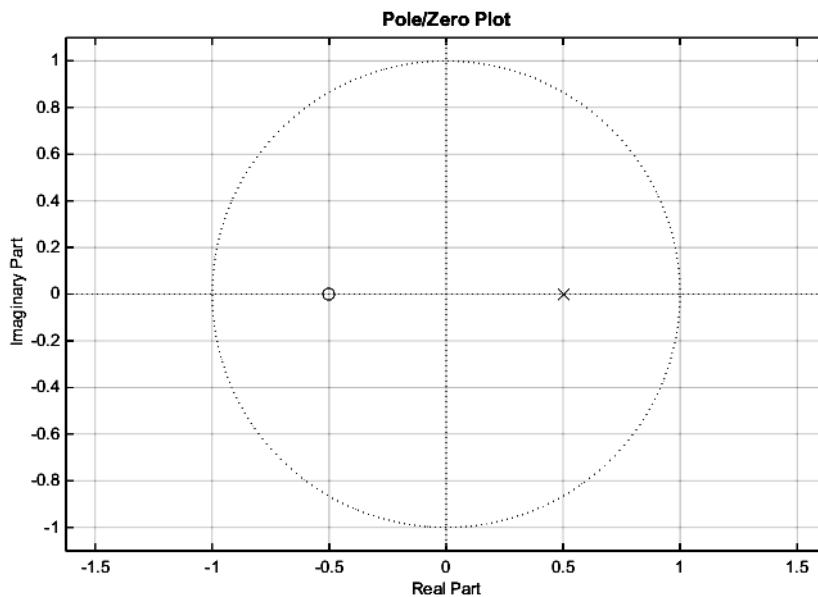
For this system, calculate the Z-transform and its region of convergence (ROC).

**[15 marks]**

- b) From the z function pole-zero diagram shown below (**Fig. Q2**):

- (i) Derive the transfer function  $H(z)$  and comment on the filter stability.

**[5 marks]**



**Fig. Q2**

- (ii) Find the unit step response impulse response of the filter  $y(n)$  for  $n=0, 1, 2, 3$  and sketch this response.

**[5 marks]**

**Total 25 marks**

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**Question 3.**

- a) Discuss the different properties of Tchebycheff, Butterworth and Bessel filters such as: frequency, time and phase responses.

**[5 marks]**

- b) Refer to **Table One**, calculate the component values for a low pass filter of order five (5). **The Butterworth** filter should have 3dB frequency of 50MHz and will be used in a  $50\Omega$  circuit. Sketch the design.

**[5 marks]****Table One**

k	n ↓	→ 2	3	4	5	6
1		1.4142	1.0000	0.7654	0.6180	0.5176
2		1.4142	2.0000	1.8478	1.6810	1.4142
3			1.0000	1.8478	2.0000	1.9319
4				0.7654	1.6810	1.9319
5					0.6810	1.4142
6						0.5176

- c) The low pass filter described in section (b) is to be converted to band –pass filter having a bandwidth of 475 MHz to 525MHz. Sketch the new design and calculate the component values.

**[10 marks]**

- d) Show how this filter can be converted or modified to become a band stop filter.

**[5 marks]****Total 25 marks****PLEASE TURN THE PAGE....**

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**Question 4.**

- a) Find the frequency response for the digital filter with the following transfer function;

$$H(z) = \frac{1 + 2z^{-1}}{1 + 5.0z^{-1} - 9.0z^{-2}}$$

**[8 marks]**

- b) Calculate the magnitude and phase if the sampling rate  $f_s$  is 20 KHz and the analogue frequency  $f$  is 4 KHz given that  $\Omega = 2\pi \frac{f}{f_s}$

**[8 marks]**

- c) Derive the difference equation.

**[4 marks]**

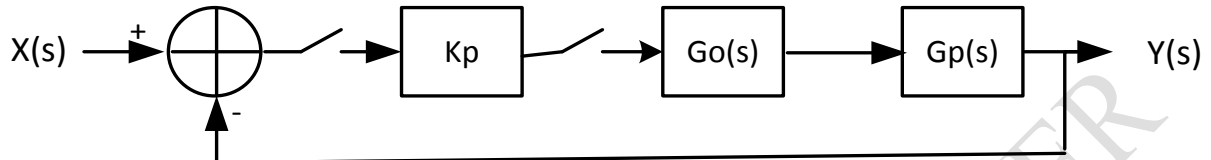
- d) Show how this difference equation could be implemented using delays and feedback.

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**Question 5.**

A block diagram for an analogue control system is shown in **Figure Q5** below:



**Figure Q5**

Where the digital controller is  $K_p$ ,

and the zero-order hold  $G_0(s) = \frac{1-e^{-sT}}{s}$ ,

with the plant  $G_p(s) = \frac{0.5}{s+0.5}$

a) Determine the closed - loop digital z transfer function for the system. **[10 marks]**

b) If the gain of the digital controller  $K_p = 10$ , determine the range of the sampling interval  $T$  that will make the closed loop stable. **[7 marks]**

c) If the sampling frequency  $f = 20$  Hz, determine the range of the controller gain  $K_p$  which will make the closed loop stable. **[8 marks]**

**Total 25 marks**

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**Question 6**

- a) Determine using the BZT method, the transfer function and difference equation for the digital filter which can replace a first order low pass resistive capacitive analogue filter. Assuming a sampling frequency of 150Hz and a cut-off frequency of 30 Hz, develop the transfer functions;

$$H_s = \frac{Y_s}{X_s} \quad \text{and} \quad H_z = \frac{Y_z}{X_z}$$

Assume  $s = \frac{T(z-1)}{2(z+1)}$  and pre-warped frequency  $W_p = \tan\left(\frac{W_c T}{2}\right)$ .

**[10 marks]**

- b) Sketch the first order low pass filter and its digital replacement with the difference equation shown.

**[10 marks]**

- c) Show how this low pass first order filter can be modified to become a high pass first order filter.

**[5 marks]****Total 25 marks****END OF QUESTIONS****Formula sheet over the page....**

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## Formula Sheet

## A Table of Basic Laplace and Z transforms

Time f (t)	Laplace F(s)	Z transforms
1. $\delta[t]$	1	1
2. $u(t)$	$\frac{1}{s}$	$\frac{z}{z-1}$
3. $t$	$\frac{1}{s^2}$	$\frac{Tz}{(z-1)^2}$
4. $e^{-at}$	$\frac{1}{s+a}$	$\frac{z}{z - e^{-aT}}$
5. $\frac{1}{1 - e^{-at}}$	$\frac{1}{s+a}$	$\frac{z - e^{-aT}}{z - e^{-aT}}$
6. $\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$	$\frac{z \sin \omega T}{z^2 - 2z \cos \omega T + 1}$
7. $\cos \omega t$	$\frac{s}{s^2 + \omega^2}$	$\frac{z^2 - z \cos \omega T}{z^2 - 2z \cos \omega T + 1}$
8. $e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$	$\frac{ze^{-aT} \sin \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$
9. $e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$	$\frac{z - e^{-aT} \cos \omega T}{z^2 - 2ze^{-aT} \cos \omega T + e^{-2aT}}$
10. $\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$	$\frac{z \sinh \omega T}{z^2 - 2z \cosh \omega T + 1}$
11. $\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$	$\frac{z \cosh \omega T}{z^2 - 2z \cosh \omega T + 1}$

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**A Table of Basic Sampled data and Z Transforms**

signal $x[n]$	z Transform $X(z)$	Region of Convergence
1 $\delta[n]$	1	all $z$
2 $u[n]$	$\frac{z}{z-1}$	$ z  > 1$
3 $\beta^n u[n]$	$\frac{z}{z-\beta}$	$ z  >  \beta $
4 $nu[n]$	$\frac{z}{(z-1)^2}$	$ z  > 1$
5 $\cos(n\Omega)u[n]$	$\frac{z^2 - z \cos \Omega}{z^2 - 2z \cos \Omega + 1}$	$ z  > 1$
6. $\sin(n\Omega)u[n]$	$\frac{z \sin \Omega}{z^2 - 2z \cos \Omega + 1}$	$ z  > 1$
7 $\beta^n \cos(n\Omega)u[n]$	$\frac{z^2 - \beta z \cos \Omega}{z^2 - 2\beta z \cos \Omega + \beta^2}$	$ z  >  \beta $
8 $\beta^n \sin(n\Omega)u[n]$	$\frac{\beta z \sin \Omega}{z^2 - 2\beta z \cos \Omega + \beta^2}$	$ z  >  \beta $

**END OF PAPER**