[ENG19]

UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) ELECTRICAL & ELECTRONICS ENGINEERING

SEMESTER ONE EXAMINATION 2018/2019

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6012

Date: Wednesday 16th January 2019 Time: 14:00 – 16:00

INSTRUCTIONS TO CANDIDATES:

There are SIX questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

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Engineering Electromagnetism
Module no. EEE6012

Question 1

A) Three symmetrical traveling waves X,Y, and Z have the following characteristics:

$v_X = 5cos4\pi x$ Volts.

Waveform	First peak value(Volts) at x=0 metre	Second peak value (Volts) at x=0.5 metre
Waveform X	5	5
Waveform Y	5	3.52
Waveform Z	5	1.01

What expression is applicable to
i) the Y wave?

[4 marks]

ii) the Z wave?

[4 marks]

B) An electromagnetic wave is propagating in the z-direction in a lossy medium with attenuation constant α=0.45 Np/m. If the wave's electric-field amplitude is 120 V/m at z=0, how far can the wave travel before its amplitude will have been reduced to
i) 12 V/m

[4 marks]

ii) 1.2 V/m

[2 marks]

iii) 1.2 μ V/m

[2 marks]

C) An RL series circuit is connected to a single phase supply with instantaneous voltage of $v_s = 20\sin(4X10^4 t - 30^o)\ Volts$. If the resistance is 3 Ω and the inductance is 0.1 mH. Find:
i) v_s in cosine form **[2 marks]**

ii) the current I in polar form **[4 marks]**

iii) the voltage across the inductance in phasor and time domains **[3 marks]**

Total 25 marks

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Question 2

A) Find the distance vector between $\mathbf{P_1}(2,2,-3)$ and $\mathbf{P_2}(-1,-2,3)$ in Cartesian coordinates and its magnitude in centimetres.

[7 marks]

B) Find the angle θ between vectors **L** and **M** of branch A using the cross product between them.

[12 marks]

C) Find the angle that vector **M** of branch A makes with the z-axis.

[6 marks]

Total 25 marks

Question 3

A) Four charges of 20 μC each are located in free space at points with Cartesian coordinates (-4,0,0), (4,0,0), (0,-2,0), and (0,4,0). Find the force on a 40 μC charge located at (0,0,3). All distances are in metres.

[12 marks]

B) A wire is formed into a square loop and placed in the x-y plane with its centre at the origin and each of its sides parallel to either the x- or y-axes. Each side is 30 cm in length, and the wire carries a current of 10 Amperes whose direction is clockwise when the loop is viewed from above. Calculate the magnetic field at the centre of the loop.

[13 marks]

Total 25 marks

Question 4

a) A 10-MHz uniform plane wave is traveling in a nonmagnetic medium with $\mu = \mu_0$ and $\varepsilon_r = 9$. Find the following parameters:

(i) the phase velocity, **[3 marks]**

(ii) the wavenumber, **[3 marks]**

(iii) the wavelength in the medium, **[3 marks]**

(iv) the intrinsic impedance of the medium. **[3 marks]**

b) The electric field phasor of a uniform plane wave traveling in a lossless medium with an intrinsic impedance of 188.5 Ω is given by $\tilde{E} = 10e^{-j4\pi y}\hat{z}$ (mV/m).

(i) Determine the associated magnetic field phasor **[5 marks]**

(ii) Find the instantaneous expression for E(y,t) if the medium is nonmagnetic ($\mu=\mu_0$).

[8 marks]

Total 25 marks

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Question 5

(a) A telephone line has the following parameters:

R= 40 Ω/m, G=400 μS/m, L=0.2 μH/m, C= 0.5 nF/m

(i) If the line operates at 10 MHz, calculate the characteristic impedance Zo and velocity u.

[10 marks]

(ii) After how many metres will the voltage drop by 30 dB in the line ?

[5 marks]

(b) Fig.5(b) shows a lossless transmission line model.

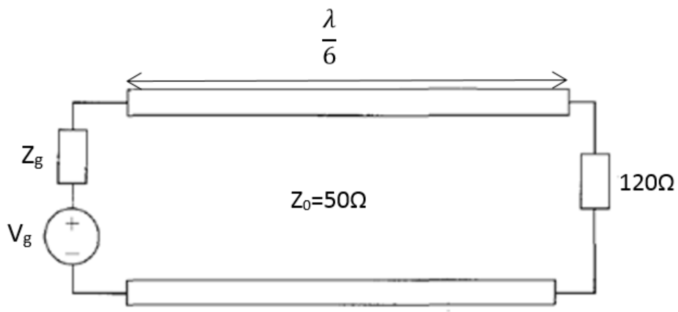

Fig.5(b) lossless transmission line model

(i) Find reflection coefficient Γ and standing wave ratio S

[5 marks]

(ii) Determine Z_{in} at the generator.

[5 marks]

Total 25 marks

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Question 6

(a) An antenna has a conical radiation pattern with a normalized radiation intensity $F(\theta) = 1$ for θ between 0^o and 45^o and zero intensity for θ between 45^o and 180^o. The pattern is independent of the azimuth angle ϕ.

(i) Find the pattern solid angle **[5 marks]**

(ii) Determine the directivity **[2 marks]**

(b) The maximum power density radiated by a short dipole at a distance of 1 km is 60 (nW/m2). If I0 = 10 A, find the radiation resistance.

[8 marks]

(c) A 3-GHz microwave link consists of two identical antennas each with a gain of 30 dB. Determine the received power, given that the transmitter output power is 1 kW and the two antennas are 10 km apart.

[5 marks]

(d) The effective area of an antenna is 9 m2. What is its directivity in decibels at 3 GHz? **[5 marks]**

Total 25 marks

END OF QUESTIONS

Formula sheet over the page….

Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\widetilde{Z}$, where $z(t) = \mathfrak{Re}\ [\widetilde{Z}e^{j\omega t}]$.

$z(t)$		$\widetilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t+\phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t+\beta x+\phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t+\beta x+\phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t+\phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\widetilde{Z}$
$\frac{d}{dt}[A\cos(\omega t+\phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\widetilde{Z}$
$\int A\sin(\omega t+\phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

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Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$ds_x = \hat{\mathbf{x}}\,dy\,dz$ $ds_y = \hat{\mathbf{y}}\,dx\,dz$ $ds_z = \hat{\mathbf{z}}\,dx\,dy$	$ds_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $ds_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $ds_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$ds_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $ds_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $ds_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2 + y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}} \cos\phi + \hat{\mathbf{y}} \sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x \cos\phi + A_y \sin\phi$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}} \cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}} \sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r \cos\phi - A_\phi \sin\phi$ $A_y = A_r \sin\phi + A_\phi \cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2 + y^2 + z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2 + y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}} \sin\theta\cos\phi + \hat{\mathbf{y}} \sin\theta\sin\phi + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}} \cos\theta\cos\phi + \hat{\mathbf{y}} \cos\theta\sin\phi - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi$	$A_R = A_x \sin\theta\cos\phi + A_y \sin\theta\sin\phi + A_z \cos\theta$ $A_\theta = A_x \cos\theta\cos\phi + A_y \cos\theta\sin\phi - A_z \sin\theta$ $A_\phi = -A_x \sin\phi + A_y \cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}} \sin\theta\cos\phi + \hat{\boldsymbol{\theta}} \cos\theta\cos\phi - \hat{\boldsymbol{\phi}} \sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}} \sin\theta\sin\phi + \hat{\boldsymbol{\theta}} \cos\theta\sin\phi + \hat{\boldsymbol{\phi}} \cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_x = A_R \sin\theta\cos\phi + A_\theta \cos\theta\cos\phi - A_\phi \sin\phi$ $A_y = A_R \sin\theta\sin\phi + A_\theta \cos\theta\sin\phi + A_\phi \cos\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2 + z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}} \sin\theta + \hat{\mathbf{z}} \cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}} \cos\theta - \hat{\mathbf{z}} \sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r \sin\theta + A_z \cos\theta$ $A_\theta = A_r \cos\theta - A_z \sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}} \sin\theta + \hat{\boldsymbol{\theta}} \cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}} \cos\theta - \hat{\boldsymbol{\theta}} \sin\theta$	$A_r = A_R \sin\theta + A_\theta \cos\theta$ $A_\phi = A_\phi$ $A_z = A_R \cos\theta - A_\theta \sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{R_{12}} \;,\; \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3} \;,\; \mathbf{E} = \frac{\mathbf{F}}{Q} ,\; \mathbf{E} = \int\frac{\rho_L dl}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \;,\; \mathbf{E} = \int\frac{\rho_S dS}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \;,\; \mathbf{E} = \int\frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n ,\; \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho \;,\; Q = \oint_S \mathbf{D}\cdot d\mathbf{S} = \int_v \rho_v dv ,\; \nabla\cdot\mathbf{D} = \rho_v ,\; W = -Q\int_A^B \mathbf{E}\cdot d\ell ,\; V_{AB} = \frac{W}{Q} = -\int_A^B \mathbf{E}\cdot d\ell ,\; V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint\mathbf{E}\cdot d\ell = 0 ,\; \nabla\times\mathbf{E} = 0 ,\; \mathbf{E} = -\nabla V ,\; W_E = \frac{1}{2}\sum_{k=1}^{n} Q_k V_k ,\; W_E = \frac{1}{2}\int\mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int\varepsilon_0 E^2 dv ,\; \mathbf{J} = \rho_v\mathbf{u} ,\; I = \int_S \mathbf{J}\cdot d\mathbf{S} ,\; \mathbf{J} = \sigma\mathbf{E} ,$$

$$R = \frac{\mathrm{V}}{\mathrm{I}} = \frac{\int\mathbf{E}\cdot d\mathbf{l}}{\int\sigma\mathbf{E}\cdot d\mathbf{S}} ,\; \mathbf{D} = \varepsilon\,\mathbf{E} ,\; \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t} ,\; E_{1t} = E_{2t} ,\; D_{1n} - D_{2n} = \rho_S ,\; D_{1n} = D_{2n} ,\; \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2 V = -\frac{\rho_v}{\varepsilon} ,\; \nabla^2 V = 0 ,\; C = \frac{Q}{V} = \frac{\varepsilon\oint\mathbf{E}\cdot d\mathbf{S}}{\int\mathbf{E}\cdot d\mathbf{l}} ,\; W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C} ,\; C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}} ,\; C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} ,\; RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12}F/m \quad ,\; \mu_o = 4\pi X10^{-7}\; H/m$$

MAGNETOSTATICS:

$$\mathbf{H}=\int_L \frac{Id\mathbf{l}\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_S \frac{\mathbf{K}dS\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_v \frac{\mathbf{J}dv\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\frac{I}{4\pi\rho}(\cos\alpha_2-\cos\alpha_1)\mathbf{a}_\phi,\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{a}_\phi=\mathbf{a}_\ell\times\mathbf{a}_\rho,$$

$$\oint\mathbf{H}\cdot d\mathbf{l}=I_{enc},\ \nabla\times\mathbf{H}=\mathbf{J},\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{H}=\frac{1}{2}\mathbf{K}\times\mathbf{a}_n,\ \mathbf{B}=\mu\mathbf{H},\ \Psi=\int_S\mathbf{B}\cdot d\mathbf{S},\ \oint\mathbf{B}\cdot d\mathbf{S}=0,\ \nabla\cdot\mathbf{B}=0,\ \mathbf{H}=-\nabla V_m,$$

$$\mathbf{B}=\nabla\times\mathbf{A},\ \mathbf{A}=\int_L\frac{\mu_0 Id\mathbf{l}}{4\pi R},\ \mathbf{A}=\int_S\frac{\mu_0\mathbf{K}dS}{4\pi R},\ \mathbf{A}=\int_v\frac{\mu_0\mathbf{J}dv}{4\pi R},\ \Psi=\oint_L\mathbf{A}\cdot d\mathbf{l},\ \mathbf{F}=Q(\mathbf{E}+\mathbf{u}\times\mathbf{B}),\ d\mathbf{F}=Id\mathbf{l}\times\mathbf{B},\ \mathbf{B}_{1n}=\mathbf{B}_{2n},$$

$$(\mathbf{H}_1-\mathbf{H}_2)\times\mathbf{a}_{n12}=\mathbf{K},\ \mathbf{H}_{1t}=\mathbf{H}_{2t},\ \frac{\tan\theta_1}{\tan\theta_2}=\frac{\mu_1}{\mu_2},\ L=\frac{\lambda}{I}=\frac{N\psi}{I},\ M_{12}=\frac{\lambda_{12}}{I_2}=\frac{N_1\psi_{12}}{I_2},\ W_m=\frac{1}{2}\int\mathbf{B}\cdot\mathbf{H}dv=\frac{1}{2}\int\mu H^2dv$$

WAVES AND APPLICATIONS:

$$V_{emf}=-\frac{d\psi}{dt}\quad,V_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\quad,\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\quad,V_{emf}=\oint_L\mathbf{E}_m\cdot d\mathbf{l}=\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$V_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}+\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}\quad,\mathbf{J}_d=\frac{\partial\mathbf{D}}{dt}\quad,\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{dt}\quad,\beta=\frac{2\pi}{\lambda},\ \underline{\gamma}=\alpha+j\beta$$

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]},\qquad\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]},\ \mathbf{E}(z,t)=E_0e^{-\alpha z}\cos(\omega t-\beta z)\mathbf{a}_x$$

$$|\underline{\eta}|=\frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}},\qquad\tan2\theta_\eta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{H}=\frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t-\beta z-\theta_\eta)\mathbf{a}_y,\ \tan\theta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{a}_E\times\mathbf{a}_H=\mathbf{a}_k$$

$$\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=120\pi\approx377\Omega,\ p(t)=\mathbf{E}\times\mathbf{H},\ p_{ave}(z)=\frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}^*{}_s),\ p_{ave}(z)=\frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta\mathbf{a}_z,\ P_{ave}=\int_S p_{ave}\cdot d\mathbf{S},$$

$$\Gamma=\frac{E_{ro}}{E_{io}}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1},\ \tau=\frac{E_{to}}{E_{io}}=\frac{2\eta_2}{\eta_2+\eta_1},\ s=\frac{|\mathbf{E}_1|_{\max}}{|\mathbf{E}_1|_{\min}}=\frac{|\mathbf{H}_1|_{\max}}{|\mathbf{H}_1|_{\min}}=\frac{1+|\Gamma|}{1-|\Gamma|},\qquad k_i\sin\theta_i=k_t\sin\theta_t,$$

$$\Gamma_\parallel=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \tau_\parallel=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \sin^2\theta_{B\parallel}=\frac{1-\mu_2\varepsilon_1/\mu_1\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2},$$

$$\Gamma_\perp=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \tau_\perp=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \sin^2\theta_{B\perp}=\frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

$\omega = \beta c$

$$S=\frac{|V_{max}|}{|V_{min}|}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma=\frac{Z_L-Z_0}{Z_L+Z_0}$$

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Antenna and Radar formula

Dipole

Solid angle:

$$\Omega_p = \iint_{4\pi} F(\theta, \phi)\, d\Omega$$

Directivity:

$$D = \frac{4\pi}{\Omega_p} \quad \text{or} \quad D = \frac{4\pi A_e}{\lambda^2}$$

Shorted dipole

$$S_0 = \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2.$$

Hertzian monopole

$$R_{rad} = 80\pi^2\left[\frac{dl}{\lambda}\right]^2$$

$$P_{rad} = \frac{1}{2} I_o^2 R_{rad}$$

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Half –wave dipole

$$\widetilde{E}_\theta = j\,60 I_0 \left\{ \frac{\cos[(\pi/2)\cos\theta]}{\sin\theta} \right\} \left(\frac{e^{-jkR}}{R} \right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}.$$

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

For Transmission line

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Distortionless line

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

Open-circuited line

$$\tilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\tilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0} \sin\beta d,$$

$$Z_{in}^{oc} = \frac{\tilde{V}_{oc}(l)}{\tilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

Short-circuited line

$$\tilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin\beta d,$$

$$\tilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0} \cos\beta d,$$

$$Z_{sc}(d) = \frac{\tilde{V}_{sc}(d)}{\tilde{I}_{sc}(d)} = jZ_0 \tan\beta d.$$

$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

$$Z_{in} = Z_o\left[\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right]$$

$$Z_{in} = Z_o\left[\frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}\right]$$

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$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g \qquad I_o = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt} G_{dr}}{4\pi} \left[\frac{\lambda}{4\pi r_1 r_2} \right]^2 \sigma P_{rad}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

$$P_{rec} = P_t G_t G_r \left(\frac{\lambda}{4\pi R} \right)^2$$

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