## UNIVERSITY OF BOLTON

# WESTERN INTERNATIONAL COLLEGE FZE <br> BENG (HONS) ELECTRICAL AND ELECTRONIC <br> ENGINEERING 

## SEMESTER ONE EXAMINATION 2018/2019

## ENGINEERING ELECTROMAGNETISM

## MODULE NO: EEE6012

Date: Wednesday 9th January 2019 Time: 10:00am-12:30pm

INSTRUCTIONS TO CANDIDATES: There are 5 questions.

Answer any 4 questions.

All questions carry equal marks.

Marks for parts of questions are shown
in brackets.

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## Q1

(a) Magnetic field vector $B$ is expressed in spherical co-ordinates as:
$B=10 / r a_{r}+r \cos \theta a_{\theta}+a_{\phi} W b / m^{2}$
Express the vector in cylindrical co-ordinates to determine $\mathrm{B}(5, \square / 2,-2)$.
(8 marks)
(b) Figure 1 shows conductors with di-electric interface half way between. Analyse the voltage drop across each dielectric in given figure, where $\varepsilon_{r 1}=2.0$ and $\varepsilon_{\mathrm{r} 2}=5.0$. The inner conductor is at $\mathrm{r}_{1}=0.02 \mathrm{~m}$ and the outer at $\mathrm{r}_{2}=0.025$ m with dielectric interface half way between.
(7 marks)


Figure 1
(c) Two extensive homogenous isotropic dielectrics meet on a plane $z=0$. For $z>0$, $\varepsilon_{r 1}=4.0$ and for $z<0, \varepsilon_{r 2}=3.0$. A uniform electric field $E_{1}$ exists for $z>=0$, where $E_{1}=5 a_{x}-2 a_{y}+3 a_{z} K V / m$. Determine:
(i) $E_{2}$ for $Z<=0$.
(4 marks)
(ii) The angles $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ make with the interface.
(2 marks)
(iii) The energy densities (in $\mathrm{J} / \mathrm{m}^{3}$ ) in both dielectrics.
(2 marks)
(iv) The energy within a cube of side $2 m$ centred at $(3,4,-5)$.

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## Q2

(a) A circular loop located on $x^{2}+y^{2}=9, z=0$ carries a direct current of 10 A along $\mathrm{a}_{\phi}$. Determine magnetic field intensity H at $(0,0,4)$ and $(0,0,-4)$.
(8 marks)
(b) Determine the self-inductance of a coaxial cable of length ' $\ell$ ' with inner radius ' $a$ ' and outer radius ' $b$ ' when inner conductor is a solid.
(10 marks)
(c) Given magnetic field intensity $\mathrm{H}_{1}=\left(-2 \mathrm{a}_{x}+6 \mathrm{a}_{y}+4 \mathrm{a}_{z}\right) \mathrm{A} / \mathrm{m}$ in region $\mathrm{y}-\mathrm{x}-2<=0$, where $\mu_{1}=5 \mu_{0}$. Calculate:
(i) Magnetization $\mathrm{M}_{1}$ and magnetic field $\mathrm{B}_{1}$.
(2 marks)
Magnetic field intensity $\mathrm{H}_{2}$ and magnetic field $\mathrm{B}_{2}$ in region $\mathrm{y}-\mathrm{x}-2>=0$, where $\mu_{2}=2 \mu_{0}$.

## Total 25 marks

## Q3

(a) The electric field (E) and magnetic field (H) in free space are given by the following expressions:

$$
E=\frac{50}{\rho} \cos \left(10^{6} t+\beta z\right) a_{\phi} V / m \quad H=\frac{H_{0}}{\rho} \cos \left(10^{6} t+\beta z\right) a_{\rho} A / m
$$

By expressing them in phasor form, determine and analyse:
i) Value of constant $\beta$ such that the fields satsify Maxwell's equations.
ii) Value of constant $\mathrm{H}_{0}$ in the given field H to satisfy Maxwell's equations.

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## Q3 continued

(b) Electric flux density $\mathbf{D}$ for a cylindrical coordinate system is shown.

$$
\vec{D}=\left(20 r+\frac{r^{2}}{3}\right) \overrightarrow{a_{r}} \text { for } r<2 \text { and } \vec{D}=\left(\frac{5 r}{138}\right) \overrightarrow{a_{r}} \text { for } r>2
$$

Evaluate charge density in both regions $\mathrm{r}<2$ and $\mathrm{r}>2$.
(c) For a homogenous medium where relative permeability $\mu_{r}=1$ and relative permittivity $\varepsilon_{r}=50$, it is given that Electric field $E=20 \square e^{j(\omega t-\beta z)} a_{x} V / m$ and magnetic field $B=\mu_{0} \mathrm{H}_{0} \mathrm{e}^{j(\omega t-\beta z)} a_{y} \mathrm{~Wb} / \mathrm{m}^{2}$. Calculate angular frequency $\omega$ and constant $\mathrm{H}_{0}$ if wavelength is 1.78 m .

## Total 25 marks

## Q4

(a) In a nonmagnetic medium $\mathrm{E}=4 \sin \left(2 \square \times 10^{7} \mathrm{t}-0.8 x\right) \mathrm{a}_{z} \mathrm{~V} / \mathrm{m}$. Find:
(i) Relative permittivity ( $\varepsilon_{r}$ ) and Intrinsic impedance $(\eta)$ and time average power carried by the wave.
(ii) Total power crossing $0.01 \mathrm{~m}^{2}$ of plane $2 \mathrm{x}+\mathrm{y}=5$.
(b) A certain transmission line 2 m long operating at $\omega=10^{6} \mathrm{rad} / \mathrm{s}$ has $\alpha=8 \mathrm{~dB} / \mathrm{m}$, $\beta=1 \mathrm{rad} / \mathrm{m}$, and $Z_{0}=60+j 40 \Omega$. If the line is connected to a source of $10<0^{0}$ $\mathrm{V}, \mathrm{Z}_{\mathrm{g}}=40 \Omega$ and terminated by a load of $20+\mathrm{j} 50 \Omega$. Determine:
(i) The input impedance.
(ii) The sending-end current.
(iii) The current at the middle of the line.

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## Q4 continued

(c) The Electric field associated with a plane wave travelling in a perfect dielectric medium having $\mu=\mu_{0}$ is given by

$$
\mathrm{E}=10 \operatorname{Cos}\left(6 \square x 10^{7} \mathrm{t}-0.4 \square \mathrm{z}\right) \mathrm{i} \mathrm{~V} / \mathrm{m}
$$

Evaluate phase velocity, permittivity of the medium and associated magnetic field H assuming velocity in free space is $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$.

Total 25 marks

Q5.
(a) Assume that a rectangular waveguide is operating in an isotropic, homogeneous dielectric with negligible magnetic properties in-$\mathrm{TM}_{13}$ mode for which $\mathrm{a}=0.015 \mathrm{~m}, \mathrm{~b}=0.008 \mathrm{~m}, \sigma=0, \mu=\mu_{0}$ and $\varepsilon=4 \varepsilon_{0}, \mathrm{H}_{\mathrm{x}}=2$ $\operatorname{Sin}(\pi x / a) \operatorname{Cos}(3 \pi y / b) \operatorname{Sin}\left(10^{11} \pi t-\beta z\right) A / m$. Determine:
(i) Cut off frequency and phase constant, $\beta$.
(ii) Propagation constant $y$ and intrinsic wave impedance $\eta$.
(b) A copper plate waveguide ( $\sigma_{\mathrm{c}}=5.8 \times 10^{7} \mathrm{~S} / \mathrm{m}$ ) operating at 4.8 GHz is supposed to deliver a minimum power of 1.2 kW to an antenna. If the guide is filled with polystyrene ( $\sigma=10^{-17} \mathrm{~S} / \mathrm{m}, \varepsilon=2.55 \varepsilon_{0}$ ) and its dimensions are $\mathrm{a}=$ 0.042 m, b $=0.026$ m, Analyse:
(i) Attenuation constants $\alpha_{d}$ and $R_{s}$ termed as real part of intrinsic impedance.
(ii) Attenuation constant $\alpha_{c}$ and Power dissipated in a length 60 cm of the guide in $\mathrm{TE}_{10}$ mode.

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## Q5 continued

(c) A fibre serves as an optical link between two cities. The step index fibre has a core diameter of $80 \mu \mathrm{~m}$, a core refractive index of 1.62 and a numerical aperture of 0.21 . Calculate:
(i) Acceptance angle, refractive index and number of modes that the fibre can propagate at a wavelength of $0.8 \mu \mathrm{~m}$.
(2 marks)
(ii) If light pulses propagate with an attenuation of $0.25 \mathrm{~dB} / \mathrm{km}$, determine distance through which the power of pulses is reduced by $40 \%$. Evaluate percentage of input power received at distance of 10 km .

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## EQUATION SHEET

## Co-ordinate systems:

$r=\sqrt{ }\left(\rho 2+z^{2}\right)$
$\theta=\tan ^{-1}(\rho / \mathrm{z})$
$\operatorname{Sin} \theta=\rho / \sqrt{ }\left(\rho 2+z^{2}\right)$
$\operatorname{Cos} \theta=z / \sqrt{ }\left(\rho 2+z^{2}\right)$

$$
\left[\begin{array}{l}
A_{\rho} \\
A_{\phi} \\
A_{z}
\end{array}\right]=\left[\begin{array}{ccc}
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1 \\
\cos \theta & -\sin \theta & 0
\end{array}\right]\left[\begin{array}{l}
A_{r} \\
A_{\theta} \\
A_{\phi}
\end{array}\right]
$$

## Capacitors:

$\mathrm{C}_{1}=\alpha \varepsilon_{0} \varepsilon_{\mathrm{r} 1} / 2.303 \log _{10}\left(\mathrm{r} / \mathrm{r}_{1}\right)$
$\mathrm{C}_{2}=\alpha \varepsilon_{0} \varepsilon_{\mathrm{r} 2} / 2.303 \log _{10}(\mathrm{r} 2 / \mathrm{r})$
$\mathrm{V}_{1}=\mathrm{VC}_{2} /\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right)$
$\mathrm{V}_{2}=\mathrm{VC}_{1} /\left(\mathrm{C}_{1}+\mathrm{C}_{2}\right)$

## Electrostatics:

Boundary conditions:
$\mathrm{E}_{1 \mathrm{t}}=\mathrm{E}_{2 \mathrm{t}}$
D1n $=D_{2 n}$
Tan $\theta_{1}=\mathrm{E}_{1 \mathrm{t}} / \mathrm{E}_{1 \mathrm{n}}$
Tan $\theta_{2}=\mathrm{E}_{2 \mathrm{t}} / \mathrm{E}_{2 n}$
Energy density, $W=(1 / 2) \varepsilon|E|^{2}$

## Magnetostatics

$\mathrm{dH}=\mathrm{Idl} x \mathrm{R} /\left(4 \pi \mathrm{R}^{3}\right)$
Ampere circuital law: $B_{1}=\mu \mathrm{I} \rho / 2 \pi \mathrm{a}^{2}$ (for $0<=\rho<=a$ )

$$
\left.\mathrm{B}_{2}=\mu \mathrm{I} / 2 \pi \rho \quad \text { (for } \mathrm{a}<=\rho<=\mathrm{b}\right)
$$

Magnetic Energy, Wm $=\mathrm{LI}^{2} / 2$
Magnetisation, $\mathrm{M}=\mathrm{X}_{\mathrm{m} 1} \mathrm{H}_{1}$
Boundary Conditions: $\mathrm{B}=\mu \mathrm{H}$

$$
\begin{aligned}
& \mathrm{H}_{1 \mathrm{n}}=\left(\mathrm{H}_{1} \cdot \mathrm{a}_{\mathrm{n}}\right) \mathrm{a}_{\mathrm{n}} \\
& \mathrm{H}_{1 \mathrm{t}}=\mathrm{H}_{2 \mathrm{t},} \quad \mathrm{~B}_{1 \mathrm{n}}=\mathrm{B}_{2 \mathrm{n}}
\end{aligned}
$$

## Maxwell's Equations

$\nabla . E_{s}=0$
$\nabla . H_{s}=0$
$\nabla x \mathrm{H}_{\mathrm{s}}=j \omega \varepsilon_{0} \mathrm{E}_{\mathrm{s}}$
$\nabla x \mathrm{E}_{\mathrm{s}}=-j \omega \mu_{0} \mathrm{H}_{\mathrm{s}}$
$\omega / \beta=C / \sqrt{ }\left(\mu_{r} \varepsilon_{r}\right)$
$E_{0} / H_{0}=\sqrt{ }\left(\mu_{0} \mu_{r} / \varepsilon_{0} \varepsilon_{r}\right)$
charge density, $\rho=\nabla \cdot \mathrm{D}=\frac{1}{r} \frac{\partial}{\partial r}\left(r \mathbf{D}_{r}\right)+\frac{1}{r} \frac{\partial \mathbf{D}_{\theta}}{\partial \theta}+\frac{\partial \mathbf{D}_{z}}{\partial z}$.

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EM wave propagation and Transmission lines
$\varepsilon_{r}=\beta^{2} /\left(\omega^{2} \mu_{0} \mu_{\mathrm{r}} \varepsilon_{0}\right)$
$\eta=\sqrt{ }(\mu / \varepsilon)$
Pavg $=E \times H$
$P_{\text {total }}=\int$ Pavg . dS
$\gamma=\alpha+j \beta$
$Z_{\text {in }}=Z_{\mathrm{o}}\left(\frac{Z_{L}+Z_{\mathrm{o}} \tanh \gamma \ell}{Z_{\mathrm{o}}+Z_{L} \tanh \gamma \ell}\right)$
$I(z=0)=\frac{V_{g}}{Z_{\text {in }}+Z_{g}}$

## Waveguides and Optical Fibres:

$$
\begin{aligned}
& f_{c_{m \times n}}=\frac{u^{\prime}}{2} \sqrt{\frac{m^{2}}{a^{2}}+\frac{n^{2}}{b^{2}}} \\
& u^{\prime}=\frac{1}{\sqrt{\mu \varepsilon}} \\
& \beta=\omega \sqrt{\mu \varepsilon} \sqrt{1-\left[\frac{f_{c}}{f}\right]^{2}} \\
& \gamma=j \beta \\
& \eta_{\mathrm{TM}_{\mathrm{mn}}}=\eta^{\prime} \sqrt{1-\left[\frac{f_{c}}{f}\right]^{2}}
\end{aligned}
$$

For the $\mathrm{TE}_{10}$ mode

$$
\alpha_{d}=\frac{\sigma \eta^{\prime}}{2 \sqrt{1-\left[\frac{f_{c}}{f}\right]^{2}}}
$$

Numerical aperture, NA $=\operatorname{Sin} \theta_{a}=\sqrt{ }\left(n_{1}{ }^{2}-n_{2}{ }^{2}\right)$
$V=\square d V\left(n_{1}{ }^{2}-n_{2}{ }^{2}\right) / \lambda$
No: of modes, $\mathrm{N}=\mathrm{V}^{2} / 2$
$\alpha \ell=10 \log _{10}[P(0) / P(\ell)]$

$$
\begin{aligned}
& V_{\mathrm{o}}=Z_{\mathrm{in}} I_{\mathrm{o}} \\
& V_{\mathrm{o}}^{+}=\frac{1}{2}\left(\mathrm{~V}_{\mathrm{o}}+Z_{\mathrm{o}} I_{\mathrm{o}}\right) \\
& V_{\mathrm{o}}^{-}=\frac{1}{2}\left(\mathrm{~V}_{\mathrm{o}}-Z_{\mathrm{o}} I_{\mathrm{o}}\right) \\
& I_{\mathrm{o}}(z=\ell / 2)=\frac{V_{\mathrm{o}}^{+}}{Z_{\mathrm{o}}} e^{-\gamma z}-\frac{V_{\mathrm{o}}^{-}}{Z_{\mathrm{o}}} e^{\gamma z} \\
& \text { phase velocity, } v=\frac{\omega}{\beta}=\frac{1}{\sqrt{\mu \varepsilon}} \\
& \nabla \times \vec{E}=-\frac{\partial \vec{B}}{\partial t}=-\mu_{0} \frac{\partial \vec{H}}{\partial t}
\end{aligned}
$$

