OCD015

UNIVERSITY OF BOLTON

OFF CAMPUS DIVISION

SWLC SRI LANKA

BENG (HONS) ELECTRICAL & ELECTRONICS ENGINEERING

SEMESTER 1 EXAMINATION AND RESIT 2018/2019

ENGINEERING ELECTROMAGNETISM

MODULE NO: EEE6002

Date: 27TH January 2019 **Time: 2hrs**

INSTRUCTIONS TO CANDIDATES:

There are six questions.

Answer ANY FOUR questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

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Q1.

(a) A series RL circuit is connected to a voltage source given by $v_s(t) = 150\ cos\omega t\ V.$ Find:

(i) The phasor current I; and **[3 marks]**

(ii) The instantaneous current i(t) for R= 400 Ω , L=3 mH, and f=31.831 kHz.

[5 marks]

(b) Find the directional derivative of V=rz^{2}cos2Φ along the direction $\widehat{\boldsymbol{A}}$=$\hat{\boldsymbol{r}}$2-$\hat{\boldsymbol{z}}$ and evaluate it at (1,90^o,2).

[8 marks]

(c) Find ∇XA at (2,0,3) in cylindrical coordinates for the vector field

$\widehat{\boldsymbol{A}}$=$\hat{\boldsymbol{r}}$10e^{-2r}cos Φ+ $\hat{\boldsymbol{z}}$10sin Φ .

[9 marks]

[Total 25 marks]

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Q2.

(a) The radii of the inner and outer conductors of a coaxial cable are 2 cm and 5 cm, respectively, and the insulating material between them has a relative permittivity of 4. The charge density on the outer conductor is 10^{-4} C/m. Calculate the total energy stored in a 40 cm length of the cable.

[10 marks]

.

(b) A 2-mm diameter copper wire with conductivity of $5.8X10^{7}$ S/m and electron mobility of 0.0032 m^2/V.s is subjected to an electric field of 20 mV/m.

Find:

I. The volume charge density of the free electrons; **[3 marks]**

II. The current density; **[3 marks]**

III. The current flowing in the wire; **[3 marks]**

IV. The electron drift velocity; and **[3 marks]**

V. The volume density of the free electrons **[3 marks]**

[Total 25 marks]

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Q3.

(a) The potential difference between two points in volts is numerically equal to the work in joules per coulomb necessary to move a coulomb of charge between the two points. A three-wire airline (balanced three-phase system) has conductors of straight cylindrical bare wires with identical radius of 12 mm and equal spacing of 2.45 m.

(i) What is the charge on each conductor?

[2 marks]

(ii) What is the voltage drop between any two conductors ?

[4 marks]

(iii) Calculate the capacitance of each wire to ground.

[4 marks]

(b) A horizontal wire with a mass per unit length of 0.2 kg/m carries a current of 4 A in the +x-direction. If the wire is placed in a uniform magnetic flux density B, what should the direction and minimum magnitude of B be in order to magnetically lift the wire vertically upward? (The acceleration due to gravity is g = **-z** 9:8 m/s^2)

[8 marks]

(c) A wire is formed into a square loop and placed in the x–y plane with its centre at the origin and each of its sides parallel to either the x- or y-axes. Each side is 40 cm in length, and the wire carries a current of 5 A whose direction is clockwise when the loop is viewed from above. Calculate the magnetic field at the centre of the loop.

[7 marks]

[Total 25 marks]

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Q4.

(a) A 10-MHz uniform plane wave is traveling in a nonmagnetic medium with $\mu = \mu_0$ and $\varepsilon_r = 9$. Find

(i) the phase velocity;

(2 marks)

(ii) the wavenumber;

(2 marks)

(iii) the wavelength in the medium;

(2 marks)

(iv) the intrinsic impedance of the medium.

(2 marks)

(b)The electric field phasor of a uniform plane wave traveling in a lossless medium with an intrinsic impedance of 188.5 Ω is given by $\widehat{\boldsymbol{E}} = \hat{\boldsymbol{z}}10\ e^{-i4\pi y}$ (mV/m).

(i) Determine the associated magnetic field phasor.

(7 marks)

(ii) Find the instantaneous expression for E(y,t) if the medium is nonmagnetic ($\mu = \mu_0$).

(10 marks)

[Total 25 marks]

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Q5.

(a)

To eliminate reflections of normally incident plane waves, a dielectric slab of thickness d and relative permittivity ε_{r2} is to be inserted between two semi-infinite media with relative permittivities $\varepsilon_{r1} = 1$ and $\varepsilon_{r3} = 16$. Use the quarter-wave transformer technique to select d and e_{r2}. Assume f = 3 GHz.

(10 marks)

(b) A 100 + j150Ω load is connected to a 75 Ω lossless line. Find:

(i) the reflection coefficient Γ;

(5 marks)

(ii) Z_{in} at 0.4 wavelength from the load;

(5 marks)

(iii) the standing wave ratio s.

(5 marks)

(Total: 25 marks)

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Q6.

(a) A monostatic radar operating at 6 GHz tracks a 0.8 m^2 target at a range of 250 m. If the gain is 40 dB, calculate the minimum transmitted power that will give a return power of 2 μW.

(10 marks)

(b) A 1-m-long dipole is excited by a 5-MHz current with an amplitude of 5 A. At a distance of 2 km, what is the power density radiated by the antenna along its broadside direction?

(7 marks)

(c)

At 100 MHz, the pattern solid angle of an antenna is 1.3 sr. Find (a) the antenna directivity D and (b) its effective area A_e.

(8 marks)

(Total: 25 marks)

END OF QUESTION

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Formula sheet

These equations are given to save short-term memorisation of details of derived equations and are given without any explanation or definition of symbols; the student is expected to know the meanings and usage.

Time-domain sinusoidal functions $z(t)$ and their cosine-reference phasor-domain counterparts $\tilde{Z}$, where $z(t) = \mathfrak{Re}\ [\tilde{Z}e^{j\omega t}]$.

$z(t)$		$\tilde{Z}$
$A\cos\omega t$	$\leftrightarrow$	A
$A\cos(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j\phi_0}$
$A\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{j(\beta x+\phi_0)}$
$Ae^{-\alpha x}\cos(\omega t + \beta x + \phi_0)$	$\leftrightarrow$	$Ae^{-\alpha x}e^{j(\beta x+\phi_0)}$
$A\sin\omega t$	$\leftrightarrow$	$Ae^{-j\pi/2}$
$A\sin(\omega t + \phi_0)$	$\leftrightarrow$	$Ae^{j(\phi_0-\pi/2)}$
$\frac{d}{dt}(z(t))$	$\leftrightarrow$	$j\omega\tilde{Z}$
$\frac{d}{dt}[A\cos(\omega t + \phi_0)]$	$\leftrightarrow$	$j\omega Ae^{j\phi_0}$
$\int z(t)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}\tilde{Z}$
$\int A\sin(\omega t + \phi_0)\,dt$	$\leftrightarrow$	$\frac{1}{j\omega}Ae^{j(\phi_0-\pi/2)}$

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Summary of vector relations.

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Coordinate variables	x, y, z	r, ϕ, z	R, θ, ϕ
Vector representation $\mathbf{A} =$	$\hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{r}}A_r + \hat{\boldsymbol{\phi}}A_\phi + \hat{\mathbf{z}}A_z$	$\hat{\mathbf{R}}A_R + \hat{\boldsymbol{\theta}}A_\theta + \hat{\boldsymbol{\phi}}A_\phi$
Magnitude of A $\quad \lvert\mathbf{A}\rvert =$	$\sqrt[+]{A_x^2 + A_y^2 + A_z^2}$	$\sqrt[+]{A_r^2 + A_\phi^2 + A_z^2}$	$\sqrt[+]{A_R^2 + A_\theta^2 + A_\phi^2}$
Position vector $\quad \overrightarrow{OP_1} =$	$\hat{\mathbf{x}}x_1 + \hat{\mathbf{y}}y_1 + \hat{\mathbf{z}}z_1$, for $P = (x_1, y_1, z_1)$	$\hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1$, for $P = (r_1, \phi_1, z_1)$	$\hat{\mathbf{R}}R_1$, for $P = (R_1, \theta_1, \phi_1)$
Base vectors properties	$\hat{\mathbf{x}}\cdot\hat{\mathbf{x}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\cdot\hat{\mathbf{y}} = \hat{\mathbf{y}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\cdot\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{z}} = \hat{\mathbf{z}}\cdot\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\phi}}$	$\hat{\mathbf{R}}\cdot\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}\cdot\hat{\boldsymbol{\phi}} = 1$ $\hat{\mathbf{R}}\cdot\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\cdot\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}\cdot\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\phi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\phi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\phi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\quad \mathbf{A}\cdot\mathbf{B} =$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\quad \mathbf{A}\times\mathbf{B} =$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\phi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length $\quad d\mathbf{l} =$	$\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{r}}\,dr + \hat{\boldsymbol{\phi}}r\,d\phi + \hat{\mathbf{z}}\,dz$	$\hat{\mathbf{R}}\,dR + \hat{\boldsymbol{\theta}}R\,d\theta + \hat{\boldsymbol{\phi}}R\sin\theta\,d\phi$
Differential surface areas	$d\mathbf{s}_x = \hat{\mathbf{x}}\,dy\,dz$ $d\mathbf{s}_y = \hat{\mathbf{y}}\,dx\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}\,dx\,dy$	$d\mathbf{s}_r = \hat{\mathbf{r}}r\,d\phi\,dz$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}\,dr\,dz$ $d\mathbf{s}_z = \hat{\mathbf{z}}r\,dr\,d\phi$	$d\mathbf{s}_R = \hat{\mathbf{R}}R^2\sin\theta\,d\theta\,d\phi$ $d\mathbf{s}_\theta = \hat{\boldsymbol{\theta}}R\sin\theta\,dR\,d\phi$ $d\mathbf{s}_\phi = \hat{\boldsymbol{\phi}}R\,dR\,d\theta$
Differential volume $dV =$	$dx\,dy\,dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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Coordinate transformation relations.

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

ELECTROSTATICS:

$$\mathbf{F}_{12} = \frac{Q_1Q_2}{4\pi\varepsilon_0R^2}\mathbf{a}_{R_{12}},\ \mathbf{F} = \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{N}\frac{Q_k(\mathbf{r}-\mathbf{r}_k)}{|\mathbf{r}-\mathbf{r}_k|^3},\ \mathbf{E} = \frac{\mathbf{F}}{Q},\ \mathbf{E} = \int\frac{\rho_L dl}{4\pi\varepsilon_0R^2}\mathbf{a}_R,\ \mathbf{E} = \int\frac{\rho_S dS}{4\pi\varepsilon_0R^2}\mathbf{a}_R,\ \mathbf{E} = \int\frac{\rho_v dv}{4\pi\varepsilon_0R^2}\mathbf{a}_R$$

$$\mathbf{E} = \frac{\rho_S}{2\varepsilon_0}\mathbf{a}_n,\ \mathbf{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\mathbf{a}_\rho,\ Q = \oint_S\mathbf{D}\cdot d\mathbf{S} = \int_v\rho_v dv,\ \nabla\cdot\mathbf{D} = \rho_v,\ W = -Q\int_A^B\mathbf{E}\cdot d\ell,\ V_{AB} = \frac{W}{Q} = -\int_A^B\mathbf{E}\cdot d\ell,\ V = \frac{Q}{4\pi\varepsilon_0 r}$$

$$\oint\mathbf{E}\cdot d\ell = 0,\ \nabla\times\mathbf{E} = 0,\ \mathbf{E} = -\nabla V,\ W_E = \frac{1}{2}\sum_{k=1}^{n}Q_kV_k,\ W_E = \frac{1}{2}\int\mathbf{D}\cdot\mathbf{E}dv = \frac{1}{2}\int\varepsilon_0E^2dv,\ \mathbf{J} = \rho_v\mathbf{u},\ I = \int_S\mathbf{J}\cdot d\mathbf{S},\ \mathbf{J} = \sigma\mathbf{E},$$

$$R = \frac{V}{I} = \frac{\int\mathbf{E}\cdot d\mathbf{l}}{\int\sigma\mathbf{E}\cdot d\mathbf{S}},\ \mathbf{D} = \varepsilon\,\mathbf{E},\ \nabla\cdot\mathbf{J} = -\frac{\partial\rho_v}{\partial t},\ E_{1t} = E_{2t},\ D_{1n} - D_{2n} = \rho_S,\ D_{1n} = D_{2n},\ \frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

$$\nabla^2V = -\frac{\rho_v}{\varepsilon},\ \nabla^2V = 0,\ C = \frac{Q}{V} = \frac{\varepsilon\oint\mathbf{E}\cdot d\mathbf{S}}{\int\mathbf{E}\cdot d\mathbf{l}},\ W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C},\ C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}},\ C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}},\ RC = \frac{\varepsilon}{\sigma}$$

$$\epsilon_o = 8.85X10^{-12}F/m\quad,\ \mu_o = 4\pi X10^{-7}\ H/m$$

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Formula sheet continued..

MAGNETOSTATICS:

$$\mathbf{H}=\int_L \frac{Id\mathbf{l}\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_S \frac{\mathbf{K}dS\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\int_v \frac{\mathbf{J}dv\times\mathbf{a}_R}{4\pi R^2},\ \mathbf{H}=\frac{I}{4\pi\rho}(\cos\alpha_2-\cos\alpha_1)\mathbf{a}_\phi,\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{a}_\phi=\mathbf{a}_\ell\times\mathbf{a}_\rho,$$

$$\oint\mathbf{H}\cdot d\mathbf{l}=I_{enc},\ \nabla\times\mathbf{H}=\mathbf{J},\ \mathbf{H}=\frac{I}{2\pi\rho}\mathbf{a}_\phi,\ \mathbf{H}=\frac{1}{2}\mathbf{K}\times\mathbf{a}_n,\ \mathbf{B}=\mu\mathbf{H},\ \Psi=\int_S\mathbf{B}\cdot d\mathbf{S},\ \oint\mathbf{B}\cdot d\mathbf{S}=0,\ \nabla\cdot\mathbf{B}=0,\ \mathbf{H}=-\nabla \mathrm{V}_m,$$

$$\mathbf{B}=\nabla\times\mathbf{A},\ \mathbf{A}=\int_L\frac{\mu_0 Id\mathbf{l}}{4\pi R},\ \mathbf{A}=\int_S\frac{\mu_0\mathbf{K}dS}{4\pi R},\ \mathbf{A}=\int_v\frac{\mu_0\mathbf{J}dv}{4\pi R},\ \Psi=\oint_L\mathbf{A}\cdot d\mathbf{l},\ \mathbf{F}=Q(\mathbf{E}+\mathbf{u}\times\mathbf{B}),\ d\mathbf{F}=Id\mathbf{l}\times\mathbf{B},\ \mathbf{B}_{1n}=\mathbf{B}_{2n},$$

$$(\mathbf{H}_1-\mathbf{H}_2)\times\mathbf{a}_{n12}=\mathbf{K},\ \mathbf{H}_{1t}=\mathbf{H}_{2t},\ \frac{\tan\theta_1}{\tan\theta_2}=\frac{\mu_1}{\mu_2},\ L=\frac{\lambda}{I}=\frac{N\psi}{I},\ M_{12}=\frac{\lambda_{12}}{I_2}=\frac{N_1\psi_{12}}{I_2},\ W_m=\frac{1}{2}\int\mathbf{B}\cdot\mathbf{H}dv=\frac{1}{2}\int\mu H^2dv$$

WAVES AND APPLICATIONS:

$$\mathrm{V}_{emf}=-\frac{d\psi}{dt}\quad,\mathrm{V}_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}\quad,\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\quad,\mathrm{V}_{emf}=\oint_L\mathbf{E}_m\cdot d\mathbf{l}=\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}$$

$$\mathrm{V}_{emf}=\oint_L\mathbf{E}\cdot d\mathbf{l}=-\int_S\frac{\partial\mathbf{B}}{\partial t}\cdot d\mathbf{S}+\oint_L(\mathbf{u}\times\mathbf{B})\cdot d\mathbf{l}\quad,\mathbf{J}_d=\frac{\partial\mathbf{D}}{dt}\quad,\nabla\times\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{dt}\quad,\beta=\frac{2\pi}{\lambda},\ \underline{\gamma}=\alpha+j\beta$$

$$\alpha=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}-1\right]},\qquad\beta=\omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1+\left[\frac{\sigma}{\omega\varepsilon}\right]^2}+1\right]},\ \mathbf{E}(z,t)=E_0e^{-\alpha z}\cos(\omega t-\beta z)\mathbf{a}_x$$

$$|\underline{\eta}|=\frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}},\qquad\tan 2\theta_\eta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{H}=\frac{E_0}{|\underline{\eta}|}e^{-\alpha z}\cos(\omega t-\beta z-\theta_\eta)\mathbf{a}_y,\ \tan\theta=\frac{\sigma}{\omega\varepsilon},\ \mathbf{a}_E\times\mathbf{a}_H=\mathbf{a}_k$$

$$\eta_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=120\pi\approx377\Omega,\ p(t)=\mathbf{E}\times\mathbf{H},\ p_{ave}(z)=\frac{1}{2}\mathrm{Re}(\mathbf{E}_s\times\mathbf{H}^*{}_s),\ p_{ave}(z)=\frac{E_0^2}{2|\underline{\eta}|}e^{-2\alpha z}\cos\theta_\eta\mathbf{a}_z,\ P_{ave}=\int_S p_{ave}\cdot d\mathbf{S},$$

$$\Gamma=\frac{E_{ro}}{E_{io}}=\frac{\eta_2-\eta_1}{\eta_2+\eta_1},\ \tau=\frac{E_{to}}{E_{io}}=\frac{2\eta_2}{\eta_2+\eta_1},\ s=\frac{|\mathbf{E}_1|_{\max}}{|\mathbf{E}_1|_{\min}}=\frac{|\mathbf{H}_1|_{\max}}{|\mathbf{H}_1|_{\min}}=\frac{1+|\Gamma|}{1-|\Gamma|},\qquad k_i\sin\theta_i=k_t\sin\theta_t,$$

$$\Gamma_\parallel=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_t-\eta_1\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \tau_\parallel=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t+\eta_1\cos\theta_i},\ \sin^2\theta_{B\parallel}=\frac{1-\mu_2\varepsilon_1/\mu_1\varepsilon_2}{1-(\varepsilon_1/\varepsilon_2)^2},$$

$$\Gamma_\perp=\frac{E_{ro}}{E_{io}}=\frac{\eta_2\cos\theta_i-\eta_1\cos\theta_t}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \tau_\perp=\frac{E_{to}}{E_{io}}=\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i+\eta_1\cos\theta_t},\ \sin^2\theta_{B\perp}=\frac{1-\mu_1\varepsilon_2/\mu_2\varepsilon_1}{1-(\mu_1/\mu_2)^2}$$

$\omega = \beta c$

$$S=\frac{|V_{max}|}{|V_{min}|}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

$$\Gamma=\frac{Z_L-Z_0}{Z_L+Z_0}$$

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Antenna and Radar formula

Hertzian monopole

$$R_{\text{rad}} = 80\pi^2 \left[\frac{dl}{\lambda}\right]^2$$

$$P_{\text{rad}} = \frac{1}{2} I_o^2 R_{\text{rad}}$$

Half –wave dipole

$$\widetilde{E}_\theta = j\,60 I_0 \left\{\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right\}\left(\frac{e^{-jkR}}{R}\right),$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}\,.$$

$$|E_{\phi s}| = \frac{\eta_o I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$|H_{\phi s}| = \frac{I_o \cos\left(\frac{\pi}{2}\cos\theta\right)}{2\pi r \sin\theta}$$

$$S_0 = \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

$$D = \frac{4\pi}{\Omega_p}, \quad A_e = \frac{\lambda^2 D}{4\pi}$$

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For a bistatic radar (one in which the transmitting and receiving antennas are separated), the power received is given by

$$P_r = \frac{G_{dt}G_{dr}}{4\pi}\left[\frac{\lambda}{4\pi r_1 r_2}\right]^2 \sigma P_{\text{rad}}$$

For a monostatic radar, $r_1 = r_2 = r$ and $G_{dt} = G_{dr}$.

For Transmission line

	Propagation Constant $\gamma = \alpha + j\beta$	Phase Velocity u_p	Characteristic Impedance Z_0
General case	$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')}$	$u_p = \omega/\beta$	$Z_0 = \sqrt{\frac{(R' + j\omega L')}{(G' + j\omega C')}}$
Lossless ($R' = G' = 0$)	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = \sqrt{L'/C'}$
Lossless coaxial	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (60/\sqrt{\varepsilon_r})\ln(b/a)$
Lossless two-wire	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120/\sqrt{\varepsilon_r}) \cdot \ln[(D/d) + \sqrt{(D/d)^2 - 1}]$ $Z_0 \simeq (120/\sqrt{\varepsilon_r})\ln(2D/d)$, if $D \gg d$
Lossless parallel-plate	$\alpha = 0,\ \beta = \omega\sqrt{\varepsilon_r}/c$	$u_p = c/\sqrt{\varepsilon_r}$	$Z_0 = (120\pi/\sqrt{\varepsilon_r})\,(h/w)$

Notes: (1) $\mu = \mu_0$, $\varepsilon = \varepsilon_r\varepsilon_0$, $c = 1/\sqrt{\mu_0\varepsilon_0}$, and $\sqrt{\mu_0/\varepsilon_0} \simeq (120\pi)\ \Omega$, where ε_r is the relative permittivity of insulating material. (2) For coaxial line, a and b are radii of inner and outer conductors. (3) For two-wire line, d = wire diameter and D = separation between wire centers. (4) For parallel-plate line, w = width of plate and h = separation between the plates.

Distortionless line

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC}$$

$$\frac{R}{L} = \frac{G}{C}, \quad Z_o = \sqrt{\frac{L}{C}}$$

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Open-circuited line

$$\widetilde{V}_{oc}(d) = V_0^+[e^{j\beta d} + e^{-j\beta d}] = 2V_0^+ \cos\beta d,$$

$$\widetilde{I}_{oc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} - e^{-j\beta d}] = \frac{2jV_0^+}{Z_0}\sin\beta d,$$

$$Z_{in}^{oc} = \frac{\widetilde{V}_{oc}(l)}{\widetilde{I}_{oc}(l)} = -jZ_0 \cot\beta l.$$

Short-circuited line

$$\widetilde{V}_{sc}(d) = V_0^+[e^{j\beta d} - e^{-j\beta d}] = 2jV_0^+ \sin\beta d,$$

$$\widetilde{I}_{sc}(d) = \frac{V_0^+}{Z_0}[e^{j\beta d} + e^{-j\beta d}] = \frac{2V_0^+}{Z_0}\cos\beta d,$$

$$Z_{sc}(d) = \frac{\widetilde{V}_{sc}(d)}{\widetilde{I}_{sc}(d)} = jZ_0 \tan\beta d.$$

$$j\omega L_{eq} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \geq 0$$

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l, \qquad \text{if } \tan\beta l \leq 0$$

For Tranmission line

$$Z_{in} = Z_o \left[\frac{Z_L + jZ_o \tan\beta\ell}{Z_o + jZ_L \tan\beta\ell}\right]$$

$$Z_{in} = Z_o \left[\frac{Z_L + Z_o \tanh\gamma\ell}{Z_o + Z_L \tanh\gamma\ell}\right]$$

$$V_o = \frac{Z_{in}}{Z_{in} + Z_g} V_g \quad , \quad I_o = \frac{V_g}{Z_{in} + Z_g}$$

$$V_o = V_L e^{j\beta l}$$

END OF FORMULA SHEET