UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING

BENG (HONS) CIVIL ENGINEERING

SEMESTER ONE EXAMINATION 2018/2019

ENGINEERING MATHEMATICS AND STRUCTURES

MODULE NO: CIE5004

Date: Friday 18th January 2019 **Time: 10:00 – 13:00**

INSTRUCTIONS TO CANDIDATES:

There are TWO Sections, A and B.

Answer Section A in ONE Answer Booklet and Section B in the other.

Section A: Q1 to Q2 (Answer ALL Questions)

Section B: Q3 to Q5 (Answer TWO Questions from three)

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Semester One Examinations 2018/2019
Engineering Mathematics and Structures
Module No: CIE5004

SECTION A: STRUCTURES - Answer ALL Questions

Question 1

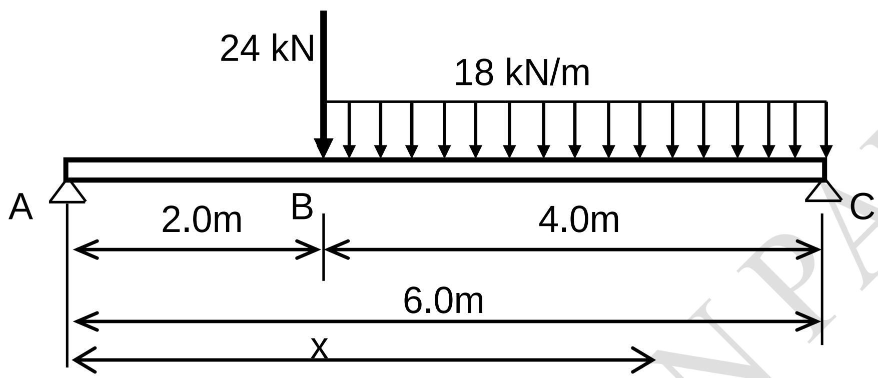

Figure Q1

Figure Q1 shows a simply supported beam with a span of 6m. The beam carries a vertical point load at B and partial uniform distributed load between B and C as shown. The vertical reaction at support A is **R_A = 40kN**.
The beam has uniform rigidity **EI = 10,000 kNm^2**

a) Write the bending moment ***M*** in terms of *x*. **(5 marks)**

b) Use the method of MaCaulay to calculate:

i. The rotation (slope) at A in radians **(7 marks)**

ii. The vertical deflection at B in mm **(7 marks)**

c) Show that the maximum deflection occurs when x = 3m and find the value of this maximum deflection in mm.

(6 marks)

Formula for the deflection of a beam: $\frac{d^2v}{dx^2} = -\frac{M}{EI}$

Total 25 marks

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Question 2

The three pin frame shown in Figure Q2 (i) is pinned to supports at A and F, with a third pin at D. The frame is subjected to a vertical point load of 100 kN at position C and a horizontal point load of 50 kN at position E.

a) Calculate the value of the support reactions at A and F. **(5 marks)**

b) Draw the axial force diagram (AFD) **(4 marks)**

c) Draw the shear force diagram (SFD) **(4 marks)**

d) Draw the bending moment diagram (BMD) **(6 marks)**

For b), c) and d) show all important values on the diagrams and produce accompanying calculations to show how these values have been derived.

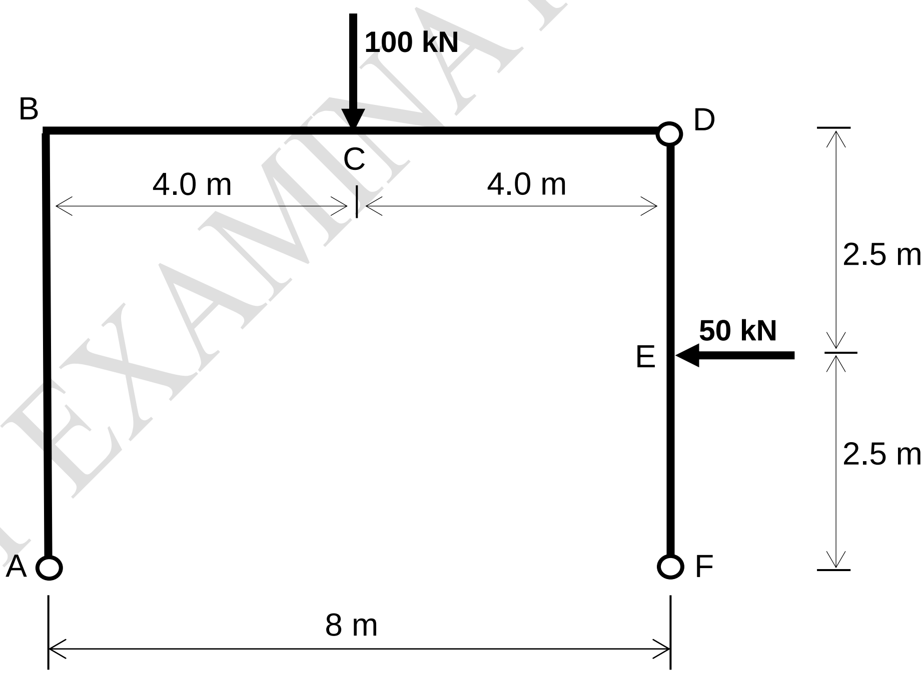

FIGURE Q2 (i)

Figure Q2 (ii) shows a very similar three pin frame, pinned to supports at A and F, with the third pin at B (no longer at D). The applied loads remain the same as Figure Q2 (i).

e) Without doing any further calculations, sketch the Bending Moment Diagram (BMD) for the three pin frame shown in Figure Q2 (ii). Do not attempt to calculate the values of the bending moments in the frame.

(6 marks)

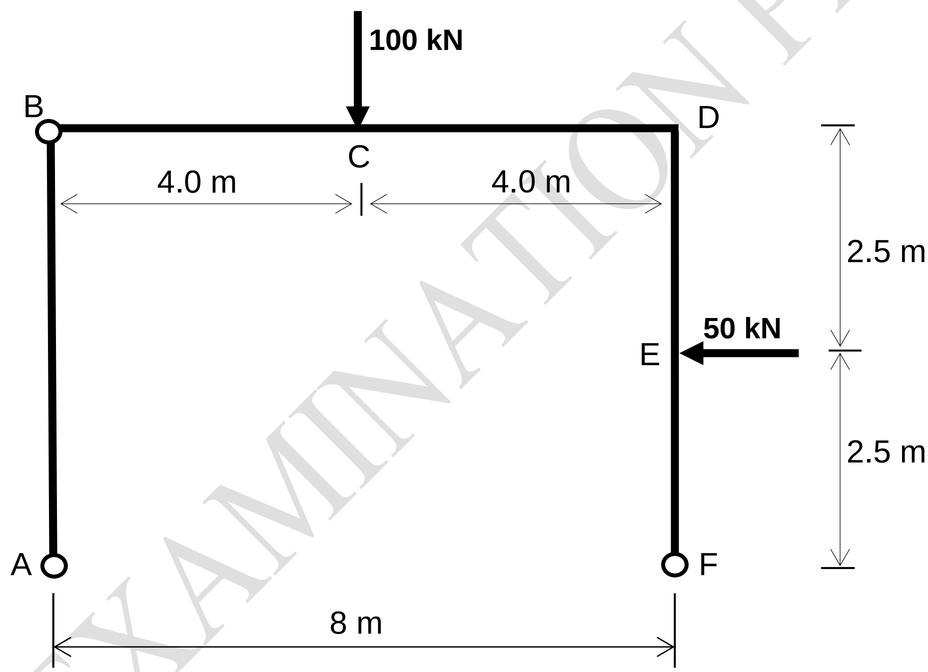

FIGURE Q2 (ii)

Total 25 marks

END OF PART A QUESTIONS

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<u>SECTION B: ENGINEERING MATHEMATICS</u> - Answer <u>TWO</u> Questions from three

<u>Question 3</u>

(a) Aggregate deliveries arriving on site follow a Poisson distribution, at an average rate of 0.8 per 10minutes. For staffing numbers, the merchant requires the hourly flow of customers. Determine the probability of the following number of customers to 3 decimal places.

(i) 3 deliveries **(3 marks)**
(ii) less than 5 deliveries **(3 marks)**
(iii) more than 5 deliveries. **(3 marks)**
(iv) the probability that one or more deliveries arrive in any half hour period. **(4 marks)**

(b) Nine concrete cubes are made from an onsite concrete mix. Extreme heat during the first two days of curing has resulted in the probability of the cubes curing too quickly and cracking being 0.29. Calculate, correct to 3 decimal places, the probability that:

(i) Exactly 4 cubes are cracked **(3 marks)**
(ii) 5 or 6 are cracked **(2 marks)**
(iii) At least 3 cubes are cracked **(4 marks)**
(iv) What is the number of cracked cubes one would expect from a sample of 16? **(3 marks)**

Total 25 marks

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Question 4

(a) The annual discharges over a reservoir's overflow weir are normally distributed with a mean flow of 14.6m^3/s and a standard deviation of 0.54m^3/s. Over the course of 365 days, calculate:

(i) The expected number of discharges ranging between 14.75 m^3/s and 15.3m^3/s.

(5 marks)

(ii) The expected number of discharges which exceed 15.63m^3/s.

(3 marks)

(iii) The expected number of discharges less than 13.25m^3/s

(3 marks)

(b) A large multi-storey car park, servicing an office block, has four exit barriers. The number of barriers in use at each of 100 instances monitored during a 24 hour period are given below

Barriers in use	0	1	2	3	4
Number of instances	10	45	31	11	3

(i) Chose an appropriate statistical model to fit to the data explaining the reasoning for your choice.

(3 marks)

(ii) Test both the goodness of fit and "too good to be true" using a 5% level of significance

(11 marks)

Total 25 marks

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Question 5

(a) The wooden cantilever element shown in Fig. Q5.1 is represented by the stiffness matrix below.

$$\begin{bmatrix} P_{XA} \\ P_{YA} \\ M_A \\ P_{XB} \\ P_{YB} \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 & -\frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & 0 & -\frac{12EI}{L^3} & \frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \times \begin{bmatrix} \delta_{XA} \\ \delta_{YA} \\ \theta_A \\ \delta_{XB} \\ \delta_{YB} \\ \theta_B \end{bmatrix}$$

(i) Write out the reduced stiffness matrix required to determine the deflection at the tip of the beam. **(9 marks)**

(ii) Determine the inverse of the stiffness matrix.

(12 marks)

(iii) Determine the tip deflection of the beam to the nearest millimetre and sketch the deflection of the beam.

(4 marks)

Question 5 continues over the page….

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Question 5 continued….

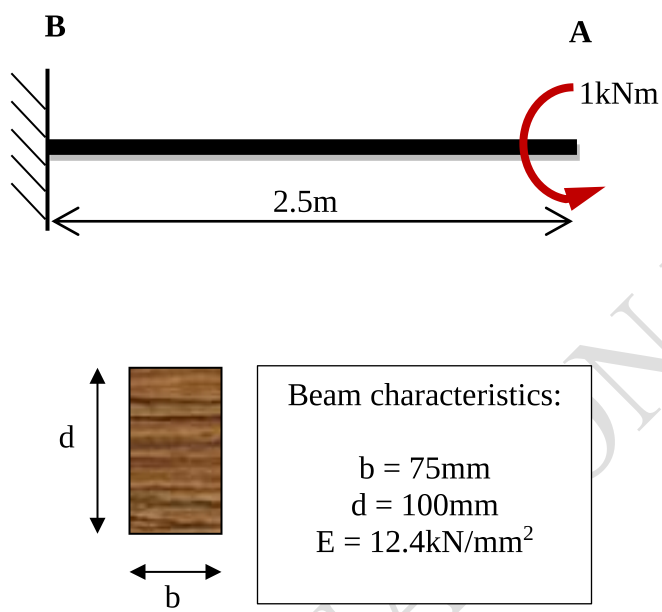


Fig. Q5.1

Total 25 marks

END OF QUESTIONS

BEng (Hons) Degree in Civil Engineering

CIE5004 – Engineering Mathematics

Formula Sheet

1. Standard Deviation

$$SD = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

2. χ^2 test

$$\chi^2 = \Sigma \frac{(O - E)^2}{E} \qquad \nu = k - m$$

3. Baye's Equation

$$Pr\,(A_k \mid E) = \frac{Pr\,(A_k) \times Pr\,(E \mid A_k)}{Pr\,(E)}$$

Standard Normal Distribution Table

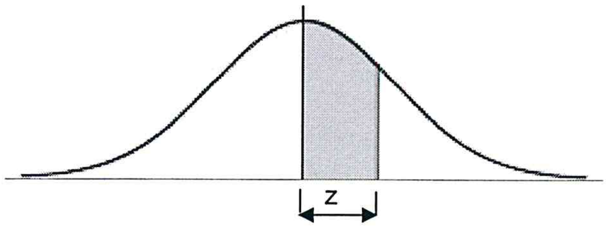
z = Number of standard deviations from mean

z	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0278	0319	0359	4	8	12	16	20	12	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	16	19	23	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	23	27	30	34
0.4	0.1554	1491	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	27	29	33
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	20	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	7	10	13	16	19	22	26	28
0.7	0.2580	2611	2642	2673	2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3032	3051	3078	3106	3133	3	6	8	11	14	17	19	22	25
0.9	0.3159	3186	3212	3238	3264	3289	3315	3340	3365	3389	3	5	8	10	13	15	18	21	23
1.0	0.3413	3438	3461	3485	3508	3531	3554	3577	3599	3621	2	5	7	9	12	14	16	18	21
1.1	0.3643	3665	3686	3708	3729	3749	3770	3790	3810	3830	2	4	6	8	11	13	15	17	19
1.2	0.3849	3869	3888	3907	3925	3944	3962	3980	3997	4015	2	4	6	7	9	11	13	15	17
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	15
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	9	10	11	13
1.5	0.4332	4345	4345	4357	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	7	8	9
1.6	0.4452	4452	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4592	4591	4599	4608	4616	4625	4633	1	2	3	4	5	5	6	7	8
1.8	0.4641	4564	4573	4592	4591	4599	4608	4616	4625	4633	1	2	3	4	5	6	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	1	1	2	2	3	3	4	4	5
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4857	0	1	1	2	2	2	3	3	4
2.2	0.4861	4865	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	1	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4940	4949	4931	4952	0	0	0	1	1	1	1	1	1
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964	0	0	0	0	1	1	1	1	1
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974	0	0	0	0	0	1	1	1	1
2.8	0.4974	4975	4976	4977	4977	4978	1979	4980	4980	4981	0	0	0	0	0	0	0	1	1
2.9	0.4981	4981	4982	4983	4983	4984	4984	4985	4986	4986	0	0	0	0	0	0	0	0	0
3.0	0.4987																		
3.1	0.4990																		
3.2	0.4993																		

Columns giving values of Pr(z) = shaded area under graph

N.B. Only the first column shows ' 0.' . In other columns, it is assumed.

Columns of mean difference in Pr(z)

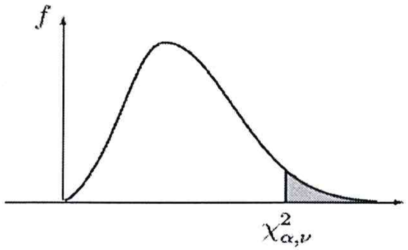


Tail Area

Z	1Tail (%)	2 Tails (%)
1.645	5	10
1.960	2.5	5
2.327	1	2
2.578	0.5	1
3.100	0.1	0.2
3.290	0.05	0.02
3.890	0.005	0.01

Percentage Points of the χ^2 Distribution

Table of χ^2 distribution for ν degrees of freedom



$\alpha =$	.995	.99	.98	.975	.95	.90	.80	.75	.70	.50	.30	.25	.29	.10	.05	.025	.02	.01	.005	.001
$\nu = 1$	$.0^3$ 393	$.0^3$ 157	$.0^3$ 628	$.0^3$ 982	.00393	.0158	.0642	.102	.148	.455	1.074	1.074	1.642	2.706	3.841	5.024	5.412	6.635	7.879	10.827
2	.0100	.0201	.0404	.506	.103	.211	.446	.575	.713	1.386	2.408	2.773	3.219	4.605	5.991	7.378	7.824	9.210	10.597	13815
3	.0717	.115	.185	.216	.352	.584	1.005	1.213	1.424	2.388	3.665	4.108	4.642	6.251	7.815	9.348	9.837	11.345	12838	18268
4	.207	.297	.429	.484	.711	1.064	1.649	1.923	2.195	3.357	4.878	5.385	5.989	7.779	9.488	11.143	11.668	13.277	14.860	18.465
5	.412	.554	.752	.831	1.145	1.610	2.343	2.65	3.000	4.351	6.004	6.626	7.289	9.236	11.070	12.832	13388	15088	16.750	20.517
6	.676	.827	1.134	1.237	1.635	2.204	3.070	3.455	3.828	5.348	7.231	7.841	8.558	10.645	12.592	14.449	15.033	16.812	18.548	22.457
7	.989	1.239	1.564	1.690	2.167	2.833	3.822	4.355	4.871	6.346	8.383	9.037	9.803	12.017	14.067	16.013	16.22	18.475	20.278	24.322
8	1.344	1.646	2.032	2.180	2.733	3.490	4.594	5.071	5.527	7.344	9.524	10.219	11.030	13.362	15.507	17.535	18.168	20.090	21.995	26.125
9	1.735	2.088	2.535	2.700	3.325	4.468	5.380	5.899	6.393	8.343	10.656	11.389	12.242	14.684	16.191	19.023	19.679	21.666	23.589	27.877
10	2.156	2.558	3.059	3.247	3.940	4.865	6.179	6.737	7.267	9.342	11.781	12.549	13.442	15.987	18.307	20.483	21.161	23.209	25.188	29.588
11	2.603	3.053	3.609	3.816	4.575	5.578	6.989	7.584	8.148	10.341	12.899	13.07	14631	17.275	19675	21.618	22.618	24.725	26.757	31.264
12	3.074	3.571	4.178	4.404	5.226	6.304	7.807	8.438	9.034	11.340	14.011	14.845	15.812	18.549	21.026	23.337	24.054	26.217	28.300	32.909
13	3.565	4.107	4.765	5.009	5.892	7.042	8.634	9.290	9.926	12.340	15.119	15.984	16.985	19.812	22.362	24.736	25.472	27.688	29.819	34.528
14	4.075	4.660	5.368	5.629	6.571	7.790	9.467	10.165	10.821	13.339	16.222	17.177	18.151	21.064	23.685	28.119	26.873	29.141	31.319	36.123
15	4.601	5.229	5.985	6.262	7.261	8.547	10.307	11.036	11.721	14.339	17.322	18.245	19.311	22.307	24.996	27.488	28.259	30.578	32.801	37.697
16	5.142	5.812	6.641	6.908	7.962	9.312	11.152	11.912	12.624	15.338	18.418	19.369	20.465	23.542	26.296	28.633	29.633	32.000	34.267	39.252
17	5.697	6.408	7.255	7.564	8.675	10.085	12.002	12.792	13.581	16.338	19.511	20.489	21.615	24.769	27.587	30.191	30.995	33.409	35.718	40.790
18	6.265	7.015	7.906	8.231	9.390	10.865	12.857	13.675	14.440	17.388	20.601	21.605	22.760	25.989	28.869	31.526	32.346	34805	37.156	42.312
19	6.844	7.633	8.567	8.907	10.117	11.651	13.716	14.562	15.352	18.33	21.689	22.718	23.204	27.204	30.144	32.852	33.687	36.191	38.582	43.820
20	7.434	8.260	9.237	9.591	10.851	12.443	14.578	15.452	16.266	19.337	22.775	23.828	25.038	28.412	31.410	34.170	35.020	37.566	39.997	45.315
21	8.034	8.897	9.915	10.283	11.591	13.240	15.445	16.344	17.182	20.337	23.858	24.935	26.171	29.615	32.671	35.479	36.343	38.392	41.401	46.797
22	8.643	9.542	10.600	10.982	12.338	14.041	16.314	17.240	18.101	21.337	24.939	26.039	27.301	33.924	36.781	37.659	40.289	42.796	48.268	48.268
23	9.250	10.195	11.293	9.260	10.196	11.688	13.091	14.848	17.187	18.137	19.021	22.337	26.018	27.141	28.429	32.007	35.172	38.076	38.968	41.638
24	9.886	10.856	11.992	12.401	13.848	15.659	18.052	19.037	19.943	23.337	27.096	28.241	29.553	33.196	36.415	39.364	40.270	42.980	45.558	51.179
25	10.520	11.524	12.697	13.120	14.611	16.473	18.940	19.939	20.807	24.337	28.172	29.339	30.675	34.382	37.652	40.646	41.566	44.314	48.928	52.620
26	11.160	12.198	13.409	13.84	15.379	17.292	19.820	20.843	21.792	25.336	29.246	30.434	31.795	35.563	38.885	41.923	42.856	45.642	48.290	54.052
27	11.808	12.879	14.125	14.125	14.125	18.114	20.703	21.749	22.719	26.336	30.319	31.528	32.912	36.741	40.133	43.194	44.140	46.963	49.645	55.476
28	12.461	13.565	14.847	15.308	18.928	18.939	21.588	22.057	23.647	27.330	31.391	32.620	31.391	37.916	41.337	44.461	45.419	48.278	50.993	58.893
29	13.121	14.256	15.574	16.047	17.708	19.768	22.475	23.567	24.577	28.336	32.461	33.711	35.139	39.087	42.557	45.722	46.693	49.588	52.336	58.302
30	13.787	14.953	16.306	16.791	18.493	20.599	23.364	24.478	25.508	29.336	33.530	34.800	36.250	40.256	43.773	46.979	47.962	50.892	53.672	59.703
40	20.706	22.164	23.834	24.838	24.433	28.509	29.051	32.345	33.660	34.872	39.335	44.165	45.616	51.805	55.795	59.342	60.436	63.691	66.766	73.402
50	27.991	29.707	31.664	32.357	34.764	37.689	41.449	42.942	44.313	49.335	54.723	56.334	58.164	63.167	67.505	71.420	72.613	76.154	79.499	86.681
60	35.535	37.485	39.699	40.482	43.188	46.459	50.641	52.294	53.809	59.335	65.227	66.981	68.927	74.397	79.082	83.298	84.580	88.379	91.952	99.607
70	43.275	45.442	47.893	48.758	51.739	55.329	59.989	61.698	63.346	69.346	75.689	77.577	79.715	85.527	90.531	95.023	96.388	100.425	104.215	112.317
80	51.171	53.539	58.539	58.213	57.153	60.391	64.278	69.207	71.145	79.334	86.120	88.130	80.405	96.578	101.880	106.629	108.069	112.329	118.321	124.839
90	59.196	61.745	64.634	65.646	69.126	73.291	78.558	80.625	82.511	89.334	96.524	98.650	101.054	107.565	113.145	118.136	199.648	124.116	128.299	137.208
100	67.327	70.065	73.142	74.222	77.929	82.358	87.945	90.133	92.129	99.334	106.006	109.141	111.667	118.498	124.342	129.561	131.142	135.807	140.170	149.44