# UNIVERSITY OF BOLTON

## SCHOOL OF ENGINEERING

## BEng (HONS) CIVIL ENGINEERING

## SEMESTER ONE EXAMINATION 2018/2019

## MATHEMATICS & STRUCTURAL ANALYSIS

## MODULE NO: CIE4011

**Date: Wednesday 16th January 2019 Time: 10:00 – 13:00**

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**INSTRUCTIONS TO CANDIDATES:**

**There are SIX questions.**

**Answer ALL SIX questions.**

**Marks for parts of questions are shown in brackets.**

**This examination paper carries a total of 100 marks.**

**Supplementary formulae sheet is provided on pages 8-9 at the rear of the question paper.**

**Lined Graph Paper is available for use.**

**All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.**

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## SECTION A: STRUCTURAL ANALYSIS

### Question 1

Figure Q1 below shows a simply supported beam with a pin support at A, and a roller support at D. The beam is carrying one vertical point load at C, and a uniform distributed load (UDL) between B and C as shown in the figure.

For the beam:

a. Determine the magnitude and direction of the support reactions at A and D. **(3 marks)**

b. Draw the Shear Force diagram. Show the values of shear force at A, B, C, and D along the beam and indicate the points along the beam where high values of bending moment will occur. **(6 marks)**

c. Draw the Bending Moment diagram, showing the values of bending moment at A, B, C, and D along the beam. **(8 marks)**

d. State the value of the maximum bending moment and its position along the beam. **(3 marks)**

**Total 20 marks**

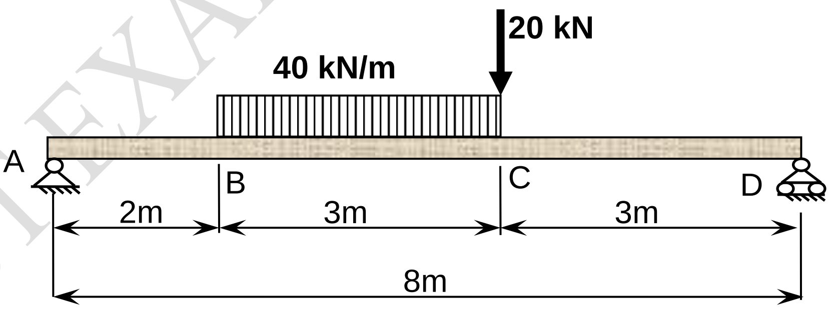


**Figure Q1**

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**Question 2**

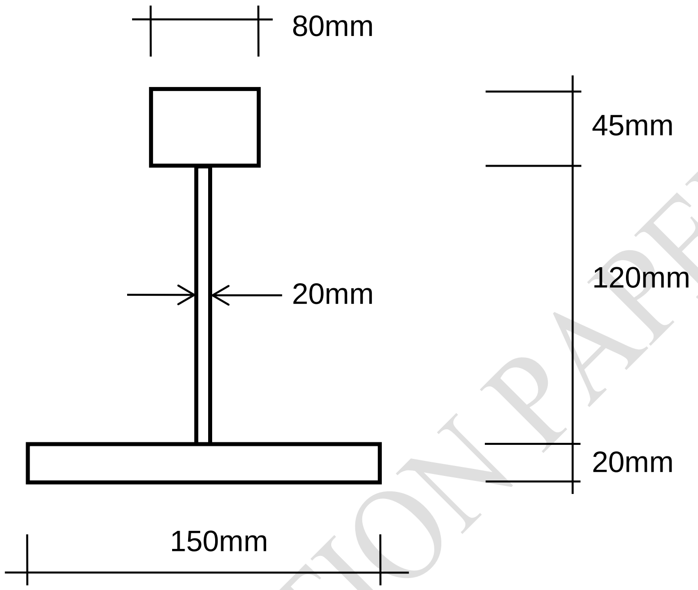


**Figure Q2 (i)**



Figure Q2 (i) shows a cross-section of an asymmetrical steel beam.

a) Determine the position of the horizontal neutral axis of the beam.

**(6 marks)**

b) What is the value of the second moment of area I about the horizontal neutral axis of the beam section?

**(9 marks)**

**Question 2 continues over the page….**

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**Question 2 continued….**

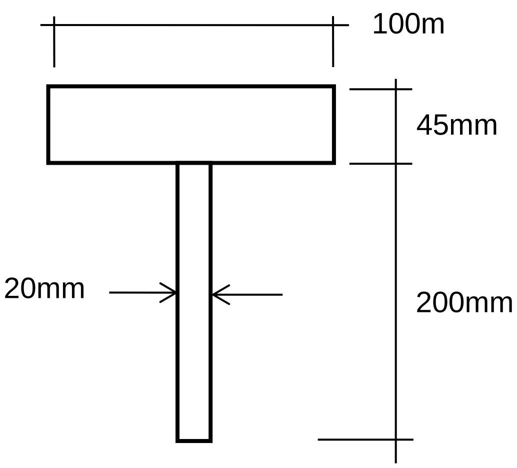


**Figure Q2 (ii)**
**Section through cantilever tee beam**

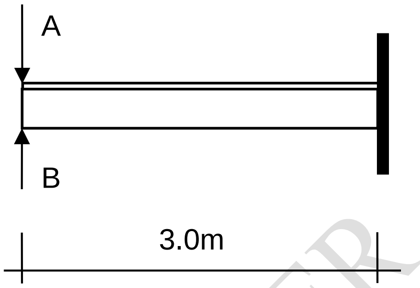


**Figure Q2 (iii)**
**Elevation on cantilever tee beam**

Figure Q2 (ii) shows a cross-section of a different asymmetrical cast iron tee beam with a cantilever span of 3.0m; also see elevation in Figure Q2 (iii). The allowable bending stresses in the tee beam are shown the table below:

| | Maximum stress (N/mm$^2$) |
|---|---|
| Tension | 25.0 |
| Compression | 100.0 |

The geometrical properties of the tee beam are shown in the table below:

| | |
|---|---|
| Distance of the horizontal neutral axis of the tee beam above the bottom of the section | 165 mm |
| Second moment of area (I) | 4587 cm$^4$ |

c) What is the maximum force A that can be applied vertically downward to the cantilever tee beam without exceeding the allowable bending stress in the tee beam (ignore force B)?

**(9 marks)**

d) What is the maximum force B that can be applied vertically upward to the cantilever tee beam without exceeding the allowable bending stress in the tee beam (ignore force A)?

**(6 marks)**

**Total 30 marks**

**END OF SECTION A**

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## SECTION B: MATHEMATICS

### Question 3

Solve the following system of simultaneous linear equations:

$$x + 3y + 2z = 5$$

$$-x + 4y + z = 8$$

$$2x + y + 3z = -5$$

**(12 marks)**

### Question 4

(a) The cooling of a building after the heating is switched off is given by the following formula:

$$T = T_0 + Ae^{-kt}$$

$T_0$ is the outdoor temperature, *T* is the temperature after *t* minutes have passed, and *A* and *k* are constants.

If the outdoor temperature is 12 degrees Celsius, and the temperature at $t = 0$ is 19 degrees Celsius, find the value of the constant *A*.

**(1 marks)**

After $t = 15$ minutes, the temperature has fallen to 18 degrees Celsius. Find the value of the constant *k*.

**(3 marks)**

Calculate the temperature after $t = 30$ minutes have passed, to the nearest tenth of a degree Celsius.

**(2 marks)**

Calculate how many minutes it will take for the temperature to fall to 15 degrees Celsius.

**(3 marks)**

(b) Using logarithms, solve the following equation:

$$3^{x+2} = 7^{2x-5}$$

**(8 marks)**

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**Question 5**

A function is defined by the following formula:

$$f(x) = x^2 - 2x$$

The value of *x* ranges between 3 and 11.

Copy and complete the following table for values of *x* and *f*(*x*):

| *x* | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|
| *f*(*x*) | | | | | | | | | |

**(1 mark)**

Let *A* be the area bounded above by the curve of the graph of the function, below by the *x*-axis, and on the left and right by the ordinates at $x = 3$ and $x = 11$.

Estimate the area *A* using:

(i) the trapezium rule with four strips **(3 marks)**

(ii) the trapezium rule with eight strips **(3 marks)**

(iii) Simpson's rule with four strips. **(3 marks)**

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**Question 6**

(a) Using Pascal's triangle, expand and simplify the following:

$$(a-2b)^5$$

**(3 marks)**

(b) Write down and simplify the first five terms of the binomial expansion for

$$\sqrt[5]{1+x}$$

Use the binomial expansion to calculate an approximate value for $\sqrt[5]{1.2}$ to four decimal places.

**(8 marks)**



**END OF QUESTIONS**

**Please turn the page (for Supplementary Formulae Sheets)….**

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## Formula sheet for structural analysis

Simply supported and cantilever beams

| Typical units | | $M_{max}$ (kNm) | $R_{support}$ (kN) | $Deflection_{max}$ (mm) |
|---|---|---|---|---|
| Simply supported beam length L | Point load P at centre | $\frac{PL}{4}$ | $\frac{P}{2}$ | $\frac{PL^3}{48EI}$ |
| Simply supported beam length L | UDL w along full length | $\frac{wL^2}{8}$ | $\frac{wL}{2}$ | $\frac{5wL^4}{384EI}$ |
| Cantilever beam length L | Point load P at tip | $PL$ | $P$ | $\frac{PL^3}{3EI}$ |
| Cantilever beam length L | UDL w along full length | $\frac{wL^2}{2}$ | $wL$ | $\frac{wL^4}{8EI}$ |

Shape properties

| Typical units | Area A ($mm^2$) | Elastic section modulus $W_{el}$ ($mm^3$) | Plastic section modulus $W_{pl}$ ($mm^3$) | 2nd moment of area I ($mm^4$) |
|---|---|---|---|---|
| Rectangle with side lengths b and h | $bh$ | $\frac{bh^2}{6}$ | $\frac{bh^2}{4}$ | $\frac{bh^3}{12}$ |

Stresses

Bending stress = $\frac{My}{I}$

Bending stress = $\frac{M}{S}$

Bending stress = $\frac{M}{z}$

Radius of gyration = $\sqrt{\frac{I}{A}}$

Axial stress = $\frac{P}{A}$

Average shear stress = $\frac{V}{A}$

Complex shapes

Centroid

$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$ and $\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$

Parallel axes theorem

$I_{xx} = \sum(I_0 + Ay^2)$

## Mathematical Formulae

### Logarithms

$$\log(ab) = \log a + \log b$$

$$\log\left(\frac{a}{b}\right) = \log a - \log b$$

$$\log(a^p) = p \log a$$

### Trapezium Rule

$$h\left(\frac{1}{2}y_0 + y_1 + y_2 + \cdots + y_{n-1} + \frac{1}{2}y_n\right)$$

### Simpson's Rule with four strips

$$\frac{1}{3}h(y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$$

### The Binomial Theorem

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \frac{n(n-1)(n-2)(n-3)}{4!}x^4 + \cdots$$



**END OF PAPER**