## UNIVERSITY OF BOLTON

## OFF CAMPUS DIVISION

## BACHELOR OF ENGINEERING (HONOURS) IN MECHANICAL ENGINEERING

## MALAYSIA - KTG

## SEMESTER 1 EXAMINATION 2018/2019

## ADVANCED MATERIALS AND STRUCTURES

## MODULE NO: AME 6002

Date: Tuesday 8 ${ }^{\text {th }}$ January 2019

INSTRUCTIONS TO CANDIDATES:

Time: 2.5 Hours

There are FOUR questions.
Answer ALL questions.
All questions carry equal marks.
Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Q1. (a) Part of a cold water cooling system as a vessel subjected to a following direct stresses in the $x, y$, and $z$ directions: 160 $\mathrm{MPa},-60 \mathrm{MPa}, 120 \mathrm{MPa}$, respectively. Due to the connection of a flange at the position of interest, two shear stresses are present, one related to $x y$ with a value of 60 MPa and another related to $y z$ with a value of 30 MPa .
(i) Draw the elemental cube showing the stresses acting.
(ii) Using this information given above and knowing that one of the Principal stresses is 110 MPa , calculate the other two principal stresses.
(iii) Evaluate the three angles ( $\alpha, \beta$, and $\gamma$ ) which describe the direction of the maximum principal stress relative to $x y z$ co-ordinate system. Produce also a sketch showing this principal stress relative the elemental cube.
(b) If the yield strength of the material in tension is 380 MPa and the material follows the von Mises yield criterion, estimate the factor of safety associated with this point in vessel.

Total 25 marks

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Q2. (a) A 300 mm internal diameter pipeline used in a chemical plant is pressurised to 3 MPa and has a blanking cover as shown schematically in Figure Q2. If it is assumed that the cover is manufactured from steel with $E=210 \mathrm{GPa}$ and $v=$ 0.31 and can be modelled as a thin flat circular plate. Hence show that:
$\frac{\delta w}{\delta r}=\frac{\frac{p r^{3}}{16}+C_{1} \frac{r}{2}+C_{2} \frac{1}{r}}{D}$
where $C_{1}$ and $C_{2}$ are constant, $p$ is the internal pressure, $r$ is the radial distance from the centre of the plate, $w$ is the normal displacement at position $r$ and $D=E t^{3} / 12\left(1-v^{2}\right)$.


Figure Q2
(b) Evaluate the necessary thickness $t$ of the cover assuming that the design stress is limited to 250 MPa .
(c) Does the thickness calculated justify the assumption? State the reason for your answer.
(d) If in the actual cover it is secured by a ring of bolts, describe how this will change the stress value and the deflected shape under the load.

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Q3. (a) Boiler plate steel is used to fabricate a cylindrical pressure vessel with a diameter $2 r$ of 5 m and a wall thickness $t$ of 8 mm with a fracture toughness of $34 \mathrm{MPa} \mathrm{m}{ }^{1 / 2}$. Inspection reveals a crack of 20 mm length running in the circumference direction.
(i) What is the maximum internal pressure $P$ allowable, assuming a safety factor of 4 ? Given the hoop stress is $P_{r} / t$ and longitudinal stress is $P_{r} / 2 t$.
(ii) Consider how the allowable maximum internal pressure would be affected if the inspection showed the crack running in the axial direction.
(iii) If the internal pressure varies between 0.8 MPa to 1.6 MPa every 40 s , how many cycles are required to extend the crack to 30 mm in the circumferential direction and how long will this take? Assume $C=10^{-}$ ${ }^{31}, Y=1.12$, and $m=3.5$.
(b)
(i) Explain the relationship between the stress intensity factor $K$ and the fracture toughness $K_{c}$ of a material.
(ii) Name two factors that $K$ is dependent upon.
(c) Sketch the graph of fatigue-crack growth rates $d a / d N$, as a function of the applied stress intensity range $K$ in metallic materials, identifying the key elements of the graph?

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Q4. (a) An endurance racing car floor panel support beam is to be fabricated from a carbon fibre reinforced polymer composite (CFRC) skins with a honeycomb core and a fibre volume fraction of 78\%.

The beam is 2.3 m long and is assumed to be fully built-in. The core is limited to a thickness of 82 mm . The loads under a worst case scenario are given in Figure Q4. Using the above information and that in Table Q4, design a suitable lay-up for the skins. Also make a simple sketch of your layup and state any assumptions you have used.
(b) Estimate the percentage weight saving if the skins had been manufactured from an aluminium alloy with a design stress of 102 MPa , an elastic modulus of 77 GPa and a relative density of 3.86 .
(c) If after tests in service the 10 kN load is observed to act 0.8 mm off left end from the $y$ - $y$ axis, explain how you would model this situation and how the lay-up would change.



Figure Q4

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Question 4 cont'd...

Table Q4

| Material | Property |  |
| :--- | :--- | :--- |
| Carbon | Elastic Modulus $E$ (GPa) | 350 |
| Fibre |  |  |
|  | Design Strain (\%) | 0.5 |
|  | In Plane Shear Strength (MPa) | 45 |
|  | Inter-Laminar Shear Strength (MPa) | 20 |
|  | Relative Density | 1.68 |
|  | Laminate Surface Bond Strength | 30 |
| Epoxy | (MPa) |  |
| Resin | Relative Density | 1.25 |
|  | Elastic Modulus $E(\mathrm{GPa})$ | 3.5 |

Total 25 marks

## END OF QUESTIONS

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## Formula Sheet

## 1. Elasticity - finding the direction vectors

$$
\begin{aligned}
& {\left[\begin{array}{l}
S_{x} \\
S_{y} \\
S_{z}
\end{array}\right]=\text { (Stress tensor) }\left(\begin{array}{c}
l \\
m \\
n
\end{array}\right)} \\
& k=\frac{1}{\sqrt{a^{2}+b^{2}+c^{2}}}
\end{aligned}
$$

where $a, b$, and $c$ are the co-factors of the eigenvalue stress tensor.

$$
\begin{array}{ll}
l=a k & l=\cos \alpha \\
m=b k & m=\cos \theta \\
n=c k & n=\cos \varphi
\end{array}
$$

## 2. Yield criterion

Von Mises:

$$
\sigma_{v m}=\frac{1}{\sqrt{2}}\left[\left(\sigma_{1}-\sigma_{2}\right)^{2}+\left(\sigma_{2}-\sigma_{3}\right)^{2}+\left(\sigma_{3}-\sigma_{1}\right)^{2}\right]^{1 / 2}
$$

Tresca:

$$
\begin{aligned}
& \sigma_{3} \geq \sigma_{2} \geq \sigma_{1} \\
& \sigma_{t r}=2 \tau_{\max }
\end{aligned}
$$

$$
\tau_{\max }=\max \left(\frac{\left|\sigma_{1}-\sigma_{2}\right|}{2} ; \frac{\left|\sigma_{1}-\sigma_{3}\right|}{2} ; \frac{\left|\sigma_{3}-\sigma_{2}\right|}{2}\right)
$$

$$
\frac{\sigma_{v m}}{\sigma_{t r}}=\frac{\sqrt{3}}{2}
$$

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## 3. Strain gauges

Transform from $x$ to $x^{\prime}$ through angle $v$ is given by:

$$
\begin{aligned}
& \varepsilon_{x \prime}=\frac{1}{2}\left(\varepsilon_{x}+\varepsilon_{y}\right)+\frac{1}{2}\left(\varepsilon_{x}-\varepsilon_{y}\right) \cos 2 \theta+\frac{\gamma_{x y}}{2} \sin 2 \theta \\
& \varepsilon_{y^{\prime}}=\frac{1}{2}\left(\varepsilon_{x}+\varepsilon_{y}\right)-\frac{1}{2}\left(\varepsilon_{x}-\varepsilon_{y}\right) \cos 2 \theta-\frac{\gamma_{x y}}{2} \sin 2 \theta \\
& \gamma_{x^{\prime} y^{\prime}}=-\left(\varepsilon_{x}-\varepsilon_{y}\right) \sin 2 \theta+\gamma_{x y} \cos 2 \theta
\end{aligned}
$$

## 4. Torsion in closed thin wall cross section (CTW)


> Shear stress $\tau$ varies inversely with thickness $t$

$$
\tau=\frac{T}{2 t A}
$$

> Shear flow
$q=\tau t$


Applied torque $T$ $T=2 q A$
$>$ Angle of twist

$$
\phi=\frac{T L}{4 A^{2} G} \oint \frac{d s}{t}
$$

## 5. Torsion in multi-cells thin wall cross-section

- Section considered as an assembly of $N$ tubular sub-sections (compartments), each subjected to torque $T_{i}$ as shown on the figure below:


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$T=\sum_{i=1}^{n} T_{i}=2 \sum_{i=1}^{n} q_{i} A_{i}$

- Common angle of twist for all compartment:

$$
\theta=\frac{L}{2 G A_{i}} \oint \frac{q_{i}-q^{\prime}}{t(s)} d s
$$

where $q$ is the shear flow of the main compartment, $q^{\prime}$ is the shear flow due to torque in adjacent compartments, $A_{i}$ is the area of crosssection $i, t$ is the thickness of the cross-section and $s$ is the circumference of the compartment.

## 6. Torsion in open thin wall cross-section (OTW)

If $b / t \geq 10$, then $\alpha=\beta=1 / 3$
and
$J_{\alpha}=J_{\beta}=J=\sum_{i=1}^{n} \frac{1}{3} b_{i} t_{i}^{3}$
Shear stress
$\tau_{\max }=\frac{T t_{\max }}{J}$
Twist angle
$\phi=\frac{L T}{G J}$

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(a)


$$
J=\frac{1}{3} b t^{3}
$$

(b)


$$
J=\frac{1}{3} b_{1} t_{1}^{3}+\frac{1}{3} b_{2} t_{2}^{3}
$$

(c)


$$
J=\frac{1}{3} b_{1} t_{1}^{3}+\frac{1}{3} b_{2} t_{2}^{3}+\frac{1}{3} b_{3} t_{3}^{3}
$$

## 7. Fracture mechanics

$Y$ values for plates loaded in tension

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(1) Through crack of length $2 a$ in an infinite plate $Y=1$.

(3) Through crack of length $2 a$ in a plate of width $w$.

$$
Y=\left(\sec \frac{\pi a}{w}\right)^{1 / 2}, \frac{2 a}{w} \leq 0.7
$$


(2) Edge crack of length $a$ in an indefinite plate $Y=1.12$.
Because plane strain and plane stress have identical stress fields, this calibration is also for an edge scratch of depth $a$ on a large body carrying tensile stress

(4) Edge crack of length $a$ in a plate of width $w$.
Y
$=0.265\left(\frac{b}{4}\right)^{4}$
$+\frac{0.875+0.265 a / w}{(a / w)^{3 / 2}}$

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(5) Through crack of length $2 a$ in a plate of width $w$.

$$
Y=\frac{2}{\pi}, a \ll D
$$



(6) Semi-elliptical surface flow.

$$
Y=\frac{1.12}{\phi^{1 / 2}}
$$

Infinite plate with a hole and symmetric double through cracks under tension.

8. Life calculations

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$$
\begin{aligned}
& \frac{d a}{d N}=C(\Delta K)^{m} \\
& N=\frac{1}{C Y^{m} \sigma_{a}^{m} \pi^{\frac{m}{2}}} \int_{a_{0}}^{a_{1}} \frac{d a}{a^{\frac{m}{2}}}
\end{aligned}
$$

## 9. Circular plates

$$
\frac{d}{d r}\left[\frac{1}{r} \frac{d}{d r}\left(r \frac{d w}{d r}\right)\right]=-\frac{Q_{r}}{D}
$$

Hooke's law is expressed in terms of $w$ as follows:

$$
\begin{aligned}
\sigma_{r} & =\frac{E}{1-v^{2}}\left(\varepsilon_{r}+v \varepsilon_{\theta}\right)=-\frac{E_{z}}{1-v^{2}}\left(\frac{d^{2} w}{d r^{2}}+\frac{v}{r} \frac{d w}{d r}\right) \\
\sigma_{\theta} & =\frac{E}{1-v^{2}}\left(\varepsilon_{\theta}+v \varepsilon_{r}\right)=-\frac{E_{z}}{1-v^{2}}\left(\frac{1}{r} \frac{d w}{d r}+v \frac{d^{2} w}{d r^{2}}\right)
\end{aligned}
$$

Bending moment and shear force:

$$
\begin{aligned}
& M_{r}=-D\left(\frac{d^{2} w}{d r^{2}}+\frac{v}{r} \frac{d w}{d r}\right), D=\frac{E t^{3}}{12\left(1-v^{2}\right)} \\
& M_{\theta}=-D\left(\frac{1}{r} \frac{d w}{d r}+v \frac{d^{2} w}{d r^{2}}\right) \\
& Q_{r}=-\frac{1}{2 \pi r} \int_{0}^{2 \pi} \int_{b}^{r} q r d r d \theta=-\frac{1}{r} \int_{b}^{r} q r d r
\end{aligned}
$$

Governing equation:
$\nabla^{4} w=\left(\frac{d^{2}}{d r^{2}}+\frac{1}{r} \frac{d}{d r}\right)\left(\frac{d^{2}}{d r^{2}}+\frac{1}{r} \frac{d}{d r}\right) w=\frac{q}{D}$

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## 10. Plastic sections

| Section | Elastic modulus | Plastic modulus | Shape factor |
| :---: | :---: | :---: | :---: |
| Rectangle | $\frac{1}{6} b h^{2}$ | $\frac{1}{4} b h^{2}$ | 1.50 |
| $\frac{\pi}{32} D^{3} \quad \frac{1}{6} D^{3}$ |  |  |  |
|  |  |  | $\pm 1.50$ |
| Thin-walled tube $\quad \frac{\pi}{4} t D^{2}$ |  |  | 1.27 |
| I-section <br> $t$ 籴 |  | $b h t+\frac{1}{4} d h^{2}$ | $\pm 1.15$ |
|  | $\frac{5}{18} t a^{2}$ | $\frac{1}{2} t a^{2}$ | 1.80 |
|  |  |  |  |

11. Composite Materials

$$
E_{\text {composite }}=E_{\text {fibre }} V_{\text {fibre }}+E_{\text {matrix }}\left(1-V_{\text {fibre }}\right)
$$

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12. Fracture toughness of some engineering materials

| Material | $K_{I C}\left(\mathrm{MN} \mathrm{~m}^{-}\right.$ | $E\left(\mathrm{GN} / \mathrm{m}^{2}\right)$ | $\mathcal{G}_{\text {IC }}\left(\mathrm{kJ} / \mathrm{m}^{2}\right)$ |
| :---: | :---: | :---: | :---: |
| Plain carbon steels | 140-200 | 200 | 100 |
| High strength steels | 30-150 | 200 | 5-110 |
| Low to medium strength steels | 10-100 | 200 | 0.5-50 |
| Titanium alloys | 30-120 | 120 | 7-120 |
| Aluminium alloys | 22-33 | 70 | 7-16 |
| Glass | 0.3-0.6 | 70 | 0.002-0.008 |
| Polycrystalline alumina | 5 | 300 | 0.08 |
| Teak-crack moves across the grain | 8 | 10 | 6 |
| Concrete | 0.4 | 16 | 1 |
| PMMA (Perspex) | 1.2 | 4 | 0.4 |
| Polystyrene | 1.7 | 3 | 0.01 |
| Polycarbonate (ductile) | 1.1 | 0.02 | 54 |
| Polycarbonate (brittle) | 0.4 | 0.02 | 6.7 |
| Epoxy resin | 0.8 | 3 | 0.2 |
| Fibreglass laminate | 10 | 20 | 5 |
| Aligned glass fibre composite - crack across fibres | 10 | 35 | 3 |
| Aligned glass fibre composite - crack down fibres | 0.3 | 10 | 0.0001 |
| Aligned carbon fibre composite - crack across fibres | 20 | 185 | 2 |

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## 13. Formulas for values of the maximum principal stresses and maximum deflection in circular plates as obtained by theory of flexure of plates

| Support and <br> loading | Principal stress <br> $\boldsymbol{\sigma}_{\max }$ | Point of <br> maximu <br> m stress | Maximum <br> deflection $\boldsymbol{w}_{\text {max }}$ |
| :--- | :--- | :--- | :--- |
| Edge simply <br> supported; load <br> uniform $\left(r_{0}=a\right)$ | $\frac{3}{8}(3+v) P \frac{a^{2}}{h^{2}}$ | Centre | $\frac{3}{16}(1-v)(5+v) \frac{p a^{4}}{E h^{3}}$ |
| Edge fixed; load <br> uniform $\left(r_{0}=a\right)$ | $\frac{3}{4} P \frac{a^{2}}{h^{2}}$ | Edge $^{\mathrm{a}}$ | $\frac{3}{16}\left(1-v^{2}\right) \frac{p a^{4}}{E h^{3}}$ |
| Edge simply <br> supported; load <br> at centre $P=$ | $\frac{3(1+v)}{2 \pi h^{2}} P\left(\frac{1}{v+1}\right.$ | Centre | $\frac{3(1-v)(5+v) P a^{2}}{4 \pi E h^{3}}$ |
| $\pi r_{0}^{2} \rho, r_{0} \rightarrow 0$, | $\left.+\ln \frac{a}{r_{0}}-\frac{1-v}{1+v} \frac{r_{0}^{2}}{4 a^{2}}\right)$ |  |  |
| but $r_{0}>0$ |  |  |  |
| Fixed edge; <br> load at centre | $\frac{3(1+v)}{2 \pi h^{2}} P\left(\ln \frac{a}{r_{0}}\right.$ | Centre | $\frac{3(1-v) P a^{2}}{4 \pi E h^{3}}$ |
| $P=\pi r_{0}^{2} p, r_{0} \rightarrow$ |  | $\left.+\frac{r_{0}^{2}}{4 a^{2}}\right)$ |  |
| 0 | $a$ must be $>1.7 r_{0}$ |  |  |

$a=$ radius of plate; $r_{0}=$ radius of centre loaded area; $h=$ thickness of plate;
$p=$ uniform load per unit area; $v=$ Poisson's ratio
${ }^{\text {a }}$ For thicker plates $(h / r>0.1)$, the deflection is $w_{\max }=C(3 / 16)(1-$ $\left.v^{2}\right)\left[\rho a^{4} / E h^{3}\right]$, where the constant $C$ depends on the ratio $h / a$ as follow: $C=$ $1+5.72(h / a)^{7}$.

## 14. Related mathematics

Cubic equations-general form
$\sigma^{3}+F_{1} \sigma^{2}+F_{2} \sigma+F_{3}=0$
where $F_{1}, F_{2}$, and $F_{3}$ are constants then the solution has three roots, say $a$, $b$, and $c$ giving:

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$$
(\sigma-a) \cdot(\sigma-b) \cdot(\sigma-c)=0
$$

Hence,
$\sigma^{3}+F_{1} \sigma^{2}+F_{2} \sigma+F_{3}=0$
as a general form.

If either $a, b$, or $c$ is known a simple quadratic equation based upon the other two unknowns can derived and solved.

Position of the Maximum moment of a propped cantilever length $L$ is given by:
$(\sqrt{2}-1) L$ from the prop end.
Finding determinants using cofactors:
Sign of cofactor

$$
\left[\begin{array}{lll}
+ & - & + \\
- & + & - \\
+ & - & +
\end{array}\right]
$$

$$
A=\left(\begin{array}{ccc}
2 & 4 & -3 \\
1 & 0 & 4 \\
2 & -1 & 2
\end{array}\right)
$$

$$
A=\left(\begin{array}{ccc}
2 & 4 & -3 \\
1 & 0 & 4 \\
2 & -1 & 2
\end{array}\right)
$$

$$
A=\left(\begin{array}{ccc}
2 & 4 & -3 \\
1 & 0 & 4 \\
2 & -1 & 2
\end{array}\right)
$$

Find determinants:
$2\left|\begin{array}{cc}0 & 4 \\ -1 & 2\end{array}\right|-4\left|\begin{array}{ll}1 & 4 \\ 2 & 2\end{array}\right|-3\left|\begin{array}{cc}1 & 0 \\ 2 & -1\end{array}\right|$
$=2[(0)(2)-(-1)(4)]-4[(1)(2)-(2)(4)]-3[(1)(-1)-(0)(2)]$
$=8+24+3$
$=35$

