UNIVERSITY OF BOLTON

SCHOOL OF ENGINEERING SCIENCES

BEng (HONS) MECHANICAL, ELECTRICAL & ELECTRONIC ENGINEERING and GERMAN COHORT

SEMESTER ONE EXAMINATIONS 2018/19

ENGINEERING MODELLING AND ANALYSIS

MODULE NO: AME5014

Date: Wednesday 16th January 2019 Time: 14:00 – 16:00

INSTRUCTIONS TO CANDIDATES:

There are EIGHT questions.

Answer ANY FIVE questions ONLY.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

Q1

The ordinary differential equation (ODE) describing the displacement x(*t*) in mm in function of time t of a voice box simulator can be modelled approximately by the equation below:

$$\ddot{x}(t) + 4\,\dot{x}(t) - 21\,x(t) = 16$$

Given: $\ddot{x}(t),\ \dot{x}(t)\ and\ x(t)\ all\ equal\ to$ 0 at t = 0,
Use the method of Laplace transforms to derive an expression for x(t) and sketch how x(t) varies with time for the first 3 seconds.

(20 marks)

Q2

It can be shown that a simple two degree of freedom electronic device in an electromagnetic field can be described by $\hat{T} = K\vec{\emptyset}$ where: $\hat{T}$ and $\vec{\emptyset}$ are torque and rotation column vectors respectively and K is the stiffness matrix. Using,

$\vec{T} = \begin{pmatrix} 50 \\ -24 \end{pmatrix}$ Nm and $K = \begin{bmatrix} 1600 & -500 \\ -500 & 1800 \end{bmatrix}$Nmm / rad

Calculate the displacement vector $\vec{\emptyset}$ in degrees.

(20 marks)

Q3

A DC motor is a first order system. Its time constant is 0.8 seconds.

a) If the speed of the DC motor is suddenly increased from being at 100 rpm into 500 rpm, what will be the speed indicated by the speedometer after 1.5 seconds?

(10 marks)

b) If the maximum speed of the motor is 1200 rpm and the motor subject a unit step input, determine the time taken for the speed output of the motor from 0 to reach 85% of its maximum speed value.

(10 marks)

Total (20 marks)

PLEASE TURN THE PAGE….

Q4

a) If $z = e^{\pi x} \sin(\pi y)$, x=1 and y=1/2, use partial differentiation to calculate:

$$\frac{\partial^2 z}{\partial y^2} + yx^2 \cdot \frac{\partial^2 z}{\partial x^2}$$

(10 marks)

b) Calculate the quantity of a resin polymer moulding by expanding gas in three dimensions (xyz) which can be expressed by the volume V bounded above by the shape $z=x^2+y^2+4$ and below by the rectangle R = {(x, y): 0 ≤ x ≤ 1, 0 ≤ y ≤ 2}.

(10 marks)

Q5

The stress σ, in MPa, at a point in a body can be described by the following matrix A relative to the global co-ordinate system xyz.

$$A = \begin{bmatrix} 0 & 5 & 7 \\ -2 & 7 & 7 \\ -1 & 1 & 4 \end{bmatrix} \text{MPa}$$

a) Using an appropriate technique, show that the principal Eigen values (principal stresses, Maximum Stresses) at this point are: λ1 = 5 MPa, λ2 = 4MPa and λ3 = 2MPa.

(10 marks)

b) Determine also the associated Eigen vector and the cosine direction of the largest principal stress.

(10 marks)

Total 20 marks

PLEASE TURN THE PAGE….

Q6

Part of a valve regular operates at a frequency ω of 1.2 rad/s. If the equation of motion is given by:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = 50Fe^{j\omega t}$$

Given: $y\ in\ mm, \quad \zeta = 0.15,\ \omega_n = 1.5\ rad/s$

(i) Derive an expression for the relationship between F and y neglecting any transient terms.

(10 marks)

(ii) Calculate the lag between y and F when F is 0.25 N.

(5 marks)

(iii) Calculate also the steady state displacement y.

(5 marks)

Total 20 marks

PLEASE TURN THE PAGE….

Q7

The energy used to cool down the temperature of an electronic device can be determined from the following integral:

$$E = V * \int_{t1}^{t2} I dt$$

Where I is the current, t is the time and V is 220 volts.

The monitored data is given below in Fig Q7. For this data determine:

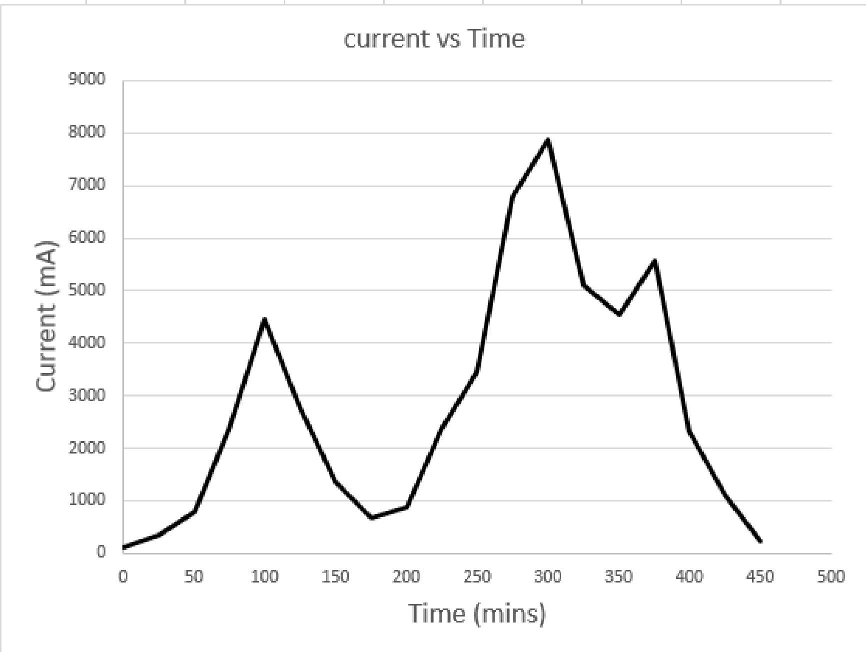
a) An estimate of the energy used in VAh (volt*Ampere*hours). **(10 marks)**

b) Estimate the time and the value at peak power consumption. **(10 marks)**

Total 20 marks

Fig Q7 Current used over time period

Time (mins)	current (mA)
0	120
25	350
50	790
75	2360
100	4450
125	2760
150	1340
175	660
200	890
225	2370
250	3450
275	6780
300	7890
325	5110
350	4530
375	5560
400	2330
425	1120
450	220



PLEASE TURN THE PAGE….

Q8

The cooling temperature of a microprocessor is modelled by Newton equation of cooling giving the temperature T in degree Celsius at time t in second. The temperature of the operating room is 19°C. The equation for cooling is given by the following 1st order differential equation:

$$\frac{dT}{dt} = -k(T - 19)$$

Given: the coefficient $k=30\text{x}10^{-5}\ s^{-1}$

a) If the temperature is 50ºC after 30mn in the room, what was the initial temperature of the object?

(11 marks)

b) Determine the time required to cool the solid object from 70ºC to 20ºC.

(9 marks)

Total 20 marks

END OF QUESTIONS

FORMULA SHEETS OVER THE PAGE….

PLEASE TURN THE PAGE….

Formula sheet

Partial Fractions

$$\frac{F(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$$

$$\frac{F(x)}{(x+a)(x+b)^2} = \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+b)^2}$$

$$\frac{F(x)}{(x^2+a)} = \frac{Ax+B}{(x^2+a)}$$

Small Changes

z = f(u, v, w)

$$\delta z \simeq \frac{\partial z}{\partial u}.\delta u + \frac{\partial z}{\partial v}.\delta v + \frac{\partial z}{\partial w}.\delta w$$

Total Differential

z = f(u, v, w)

$$dz = \frac{\partial z}{\partial u}du + \frac{\partial z}{\partial v}dv + \frac{\partial z}{\partial w}dw$$

Rate of Change

z = f(u, v, w)

$$\frac{dz}{dt} = \frac{\partial z}{\partial u}.\frac{du}{dt} + \frac{\partial z}{\partial v}.\frac{dv}{dt} + \frac{\partial z}{\partial w}.\frac{dw}{dt}$$

Eigenvalues

$$|A - \lambda \mathrm{I}| = 0$$

PLEASE TURN THE PAGE….

Eigenvectors

$$(A-\lambda_r \mathrm{I})x_r = 0$$

Integration

$$\int u.\frac{dv}{dx}dx = uv - \int v.\frac{du}{dx}dx$$

Fourier

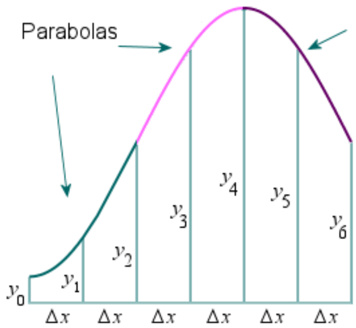
$$F(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\cos nx\, dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\sin nx\, dx$$

Simpson's rule
To calculate the area under the curve which is the integral of the function **Simpson's Rule** is used as shown in the figure below:



The area into *n* equal segments of width *Δx*. Note that in Simpson's Rule, *n* must be EVEN. The approximate area is given by the following rule:

$$Area = \int_a^b f(x)dx = \frac{\Delta x}{3}(y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 \dots + 4y_{n-1} + y_n)$$

Where $\Delta x = \frac{b-a}{n}$

Differential equation

Homogeneous form:

$$a\ddot{y} + b\dot{y} + cy = 0$$

Characteristic equation:

$$a\lambda^2 + b\lambda + c = 0$$

Quadratic solutions :

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i. If $b^2 - 4ac > 0$, λ_1 and λ_2 are distinct real numbers then the general solution of the differential equation is:

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

A and B are constants.

ii. If $b^2 - 4ac = 0$, $\lambda_1 = \lambda_2 = \lambda$ then the general solution of the differential equation is:

$$y(t) = e^{\lambda t}(A + Bx)$$

A and B are constants.

iii. If $b^2 - 4ac < 0$, λ_1 and λ_2 are complex numbers then the general solution of the differential equation is:

$$y(t) = e^{\alpha t}[Acos(\beta t) + Bsin(\beta t)]$$

$$\alpha = \frac{-b}{2a} \quad and \quad \beta = \frac{\sqrt{4ac - b^2}}{2a}$$

A and B are constants.

PLEASE TURN THE PAGE….

Inverse of 2x2 matrices:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

The inverse of A can be found using the formula:

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

modelling growth and decay of engineering problem

$C(t) = C_0 e^{kt}$

$k > 0$ gives exponential growth

$k < 0$ gives exponential decay

First order system

$$y(t) = k(1 - e^{-\frac{t}{\tau}})$$

Transfer function: $\dfrac{k}{\tau s+1}$

PLEASE TURN THE PAGE….

<u>Derivatives table:</u>

$y = f(x)$	$\frac{dy}{dx} = f'(x)$
k, any constant	0
x	1
x^2	$2x$
x^3	$3x^2$
x^n, any constant n	nx^{n-1}
e^x	e^x
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\sin kx$	$k \cos kx$
$\cos x$	$-\sin x$
$\cos kx$	$-k \sin kx$
$\tan x = \frac{\sin x}{\cos x}$	$\sec^2 x$
$\tan kx$	$k \sec^2 kx$
$\operatorname{cosec} x = \frac{1}{\sin x}$	$-\operatorname{cosec} x \cot x$
$\sec x = \frac{1}{\cos x}$	$\sec x \tan x$
$\cot x = \frac{\cos x}{\sin x}$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$

PLEASE TURN THE PAGE….

Integral table:

$f(x)$	$\int f(x)\,dx$
k, any constant	$kx + c$
x	$\frac{x^2}{2} + c$
x^2	$\frac{x^3}{3} + c$
x^n	$\frac{x^{n+1}}{n+1} + c$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$
e^x	$e^x + c$
e^{kx}	$\frac{1}{k}e^{kx} + c$
$\cos x$	$\sin x + c$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin x$	$-\cos x + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
$\tan x$	$\ln(\sec x) + c$
$\sec x$	$\ln(\sec x + \tan x) + c$
$\mathrm{cosec}\, x$	$\ln(\mathrm{cosec}\, x - \cot x) +$
$\cot x$	$\ln(\sin x) + c$
$\cosh x$	$\sinh x + c$
$\sinh x$	$\cosh x + c$
$\tanh x$	$\ln\cosh x + c$
$\coth x$	$\ln\sinh x + c$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$

Laplace table:

$f(t)$	$F(s)$		$f(t)$	$F(s)$
1	$\frac{1}{s}$		$u_c(t)$	$\frac{e^{-cs}}{s}$
t	$\frac{1}{s^2}$		$\delta(t)$	1
t^n	$\frac{n!}{s^{n+1}}$		$\delta(t-c)$	e^{-cs}
e^{at}	$\frac{1}{s-a}$		$f'(t)$	$sF(s)-f(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$		$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
$\cos bt$	$\frac{s}{s^2+b^2}$		$(-t)^n f(t)$	$F^{(n)}(s)$
$\sin bt$	$\frac{b}{s^2+b^2}$		$u_c(t)f(t-c)$	$e^{-cs}F(s)$
$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$		$e^{ct}f(t)$	$F(s-c)$
$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$		$\delta(t-c)f(t)$	$e^{-cs}f(c)$

END OF THE FORMULA SHEET