## UNIVERSITY OF BOLTON

## WESTERN INTERNATIONAL COLLEGE FZE

BENG (HONS) MECHANICAL ENGINEERING
SEMESTER ONE EXAMINATION 2018/2019
ENGINEERING MODELLING AND ANALYSIS
MODULE NO: AME5014

Date: Wednesday 9th January 2019

INSTRUCTIONS TO CANDIDATES:

CANDIDATES REQUIRE:

Time: 2:00PM - 4:00PM

There are FIVE questions on this paper

Answer ANY FOUR questions only
All questions carry equal marks
Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleaned prior to the examination.

Formula Sheet (attached)

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## Question 1

i. Determine the Eigen values and Eigenvectors for the matrix:

$$
\left[\begin{array}{cc}
5 & -2 \\
-9 & 2
\end{array}\right]
$$

(15 marks)
ii. Obtain a fourier series for the periodic function $f(x)$ defines as
$F(x)=-k$, when $-\pi<x<0$
$F(x)=+k$, when $0<x<\pi$
The function is periodic outside of this range with period $2 \pi$
(10 marks)

## Total 25 marks

## Question 2

A large marquee is to be made in the form of a rectangular box-like shape with canvas covering on the top, back and sides, if the volume of the marquee is to be $250 \mathrm{~m}^{3}$.

Determine the following
a) Sketch the open rectangular box with proper dimensions.
b) Find the dimensions of the open rectangular box
c) Show that the total surface area is minimum
d) Determine the minimum surface area of canvas

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## Question 3.

i. The attitude of employees towards new company policy is tabulated below. The employees are grouped according to their job description:
Mechanical engineer, programmer or systems engineer.

| Attitude | Mechanical <br> engineer | Programmer | System <br> Engineer |
| :--- | :--- | :--- | :--- |
| Like | 46 | 168 | 196 |
| Indifferent | 100 | 572 | 1148 |
| Dislike | 32 | 248 | 1076 |

Use a Chi Square ( $\chi^{2}$ ) test to check the hypothesis that, there are variations in attitude depending on job description using a $5 \%$ level of significance.
ii. Two curves $y=x^{2}+1$ and $y=7-x$ are such that both the curves intersect each other. Determine the following.
a) The points of intersection of both the curves.
b) Provide a table for both the curves showing all the points.
c) Sketch the curves.
d) Find the area between the curves.

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## Question 4.

i. Find the Laplace transforms of:
(a) $4 \sin (a t+b)$
ii. The current flowing in an electrical circuit is given by differential equation $\mathrm{Ri}+\mathrm{L} \frac{d i}{d t}=\mathrm{E}$

Where $E, L$ and $R$, are constants use Laplace transforms to solve the equation for current (i), given that when $t=0, i=0$.
iii. Determine the inverse laplace transform of

$$
\frac{4 s-5}{s 2-s-2}
$$

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## Question 5

1) The pressure $p$, volume $V$ and temperature $T$ of a gas are related by $p V=k T$, where k is a constant.
Determine the total differentials
(a) dp and
(6 marks)
(b) dT
(6 marks)
in terms of $\mathrm{p}, \mathrm{V}$ and T .
2) Determine the rate of increase of diagonal AC of the rectangular solid, shown in

Figure 1 below, correct to 2 significant figures, if the sides $x, y$ and $z$ increase at $6 \mathrm{~mm} / \mathrm{s}, 5 \mathrm{~mm} / \mathrm{s}$ and $4 \mathrm{~mm} / \mathrm{s}$ when these three sides are $5 \mathrm{~cm}, 4 \mathrm{~cm}$ and 3 cm respectively.


Figure 1: Rectangular Solid.

## END OF QUESTIONS

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## FORMULA SHEET

## Laplace transforms

| (i) 1 | $\frac{1}{s}$ |
| :---: | :---: |
| (ii) $k$ | $\frac{k}{s}$ |
| (iii) $\mathrm{e}^{\mathrm{at}}$ | $\frac{1}{s-a}$ |
| (iv) $\sin a t$ | $\frac{a}{s^{2}+a^{2}}$ |
| (v) $\cos a t$ | $\frac{s}{s^{2}+a^{2}}$ |
| (vi) $t$ | $\frac{1}{s^{2}}$ |
| (vii) $t^{2}$ | $\frac{2!}{s^{3}}$ |
| (viii) $t^{n}(n=1,2,3, \ldots)$ | $\frac{n!}{s^{n+1}}$ |
| (ix) $\cosh a t$ | $\frac{s}{s^{2}-a^{2}}$ |
| (x) $\sinh a t$ | $\frac{a}{s^{2}-a^{2}}$ |
| (i) $\mathrm{e}^{a t} t^{n}$ | $\frac{n!}{(s-a)^{n+1}}$ |
| (ii) $\mathrm{e}^{a t} \sin \omega t$ | $\frac{\omega}{(s-a)^{2}+\omega^{2}}$ |
| (iii) $\mathrm{e}^{a t} \cos \omega t$ | $\frac{s-a}{(s-a)^{2}+\omega^{2}}$ |
| (iv) $\mathrm{e}^{a t} \sinh \omega t$ | $\frac{\omega}{(s-a)^{2}-\omega^{2}}$ |
| (v) $\mathrm{e}^{a t} \cosh \omega t$ | $\frac{s-a}{(s-a)^{2}-\omega^{2}}$ |

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Inverse laplace.

| (i) | $\frac{1}{s}$ | 1 |
| :---: | :---: | :---: |
| (ii) | $\frac{k}{s}$ | $k$ |
| (iii) | $\frac{1}{s-a}$ | $\mathrm{e}^{a t}$ |
| (iv) | $\frac{a}{s^{2}+a^{2}}$ | $\sin a t$ |
| (v) | $\frac{s}{s^{2}+a^{2}}$ | $\cos a t$ |
| (vi) | $\frac{1}{s^{2}}$ | $t$ |
| (vii) | $\frac{2!}{s^{3}}$ | $t^{2}$ |
| (viii) | $\frac{n!}{s^{n+1}}$ | $t^{n}$ |
| (ix) | $\frac{a}{s^{2}-a^{2}}$ | sinh at |
| (x) | $\frac{s}{s^{2}-a^{2}}$ | cosh $a t$ |
| (xi) | $\frac{n!}{(s-a)^{n+1}}$ | $\mathrm{e}^{a t} t^{n}$ |
| (xii) | $\frac{\omega}{(s-a)^{2}+\omega^{2}}$ | $\mathrm{e}^{a t} \sin \omega t$ |
| (xiii) | $\frac{s-a}{(s-a)^{2}+\omega^{2}}$ | $\mathrm{e}^{a t} \cos \omega t$ |
| (xiv) | $\frac{\omega}{(s-a)^{2}-\omega^{2}}$ | $\mathrm{e}^{a t} \sinh \omega t$ |
| (xv) | $\frac{s-a}{(s-a)^{2}-\omega^{2}}$ | $\mathrm{e}^{a t} \cosh \omega t$ |

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## Maxima /Minima

$Z=F(x, y)$
Stationary Points $\quad \frac{\partial Z}{\partial x}=0, \quad \frac{\partial Z}{\partial y}=0$
$\Delta=\left(\frac{\partial^{2} z}{\partial x \partial y}\right)^{2}-\left(\frac{\partial^{2} z}{\partial x^{2}}\right)\left(\frac{\partial^{2} z}{\partial y^{2}}\right)$

## Statistics

Chi-square distribution

$$
\chi^{2}=\sum \frac{\left(O_{i}-E_{i}\right)^{2}}{E_{i}}
$$

## Partial Fractions

$\frac{F(x)}{(x+a)(x+b)}=\frac{A}{(x+a)}+\frac{B}{(x+b)}$
$\frac{F(x)}{(x+a)(x+b)^{2}}=\frac{A}{(x+a)}+\frac{B}{(x+b)}+\frac{C}{(x+b)^{2}}$
$\frac{F(x)}{\left(x^{2}+a\right)}=\frac{A x+B}{\left(x^{2}+a\right)}$

## Eigenvalues

$|A-\lambda I|=0$

## Eigenvectors

$\left(A-\lambda_{r} \mathrm{I}\right) x_{r}=0$

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Total differential, rates of change and small changes

$$
\begin{aligned}
& \mathrm{d} z=\frac{\partial z}{\partial u} \mathrm{~d} u+\frac{\partial z}{\partial v} \mathrm{~d} v+\frac{\partial z}{\partial w} \mathrm{~d} w+\cdots \\
& \frac{\mathrm{d} z}{\mathrm{~d} t}=\frac{\partial z}{\partial \mathrm{u}} \frac{\mathrm{~d} u}{\mathrm{~d} t}+\frac{\partial z}{\partial v} \frac{\mathrm{~d} v}{\mathrm{~d} t}+\frac{\partial z}{\partial w} \frac{\mathrm{~d} w}{\mathrm{~d} t}+\cdots \\
& \delta z \approx \frac{\partial z}{\partial u} \delta u+\frac{\partial z}{\partial v} \delta v+\frac{\partial z}{\partial w} \delta w+\cdots \\
& \mathrm{L}\{x\}=\bar{x} \\
& \mathrm{~L}\{\dot{x}\} \doteq s \bar{x}-x_{0} \\
& \mathrm{~L}\{\ddot{x}\}=s^{2} \bar{x}-\mathrm{s} x_{0}-x_{1}
\end{aligned}
$$

## Fourier Series

$$
\begin{aligned}
& a_{0}=\frac{1}{2 \pi} \int_{-\pi}^{\pi} f(x) \mathrm{d} x \\
& a_{n}=\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos n x \mathrm{~d} x \\
& b_{n}=\frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin n x \mathrm{~d} x \\
& f(x)=a_{0}+\sum_{n=1}^{\infty}\left(a_{n} \cos n x+b_{n} \sin n x\right)
\end{aligned}
$$

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Chi-Square Distribution Table


The shaded area is equal to $\alpha$ for $\chi^{2}=\chi_{\alpha}^{2}$.

| df | $\chi_{.995}^{2}$ | $\chi .990$ | $\chi^{2} .975$ | $\chi .950$ | $\chi_{\text {2 }}{ }^{2}{ }^{\text {a }}$ | $\chi .100$ | $\chi .050$ | $\chi{ }^{2}{ }^{2}{ }^{2} 5$ | $\chi{ }^{2}{ }^{2} 10$ | $\chi .{ }^{2}{ }^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.000 | 0.000 | 0.001 | 0.004 | 0.016 | 2.706 | 3.841 | 5.024 | 6.635 | 7.879 |
| 2 | 0.010 | 0.020 | 0.051 | 0.103 | 0.211 | 4.605 | 5.991 | 7.378 | 9.210 | 10.597 |
| 3 | 0.072 | 0.115 | 0.216 | 0.352 | 0.584 | 6.251 | 7.815 | 9.348 | 11.345 | 12.838 |
| 4 | 0.207 | 0.297 | 0.484 | 0.711 | 1.064 | 7.779 | 9.488 | 11.143 | 13.277 | 14.860 |
| 5 | 0.412 | 0.554 | 0.831 | 1.145 | 1.610 | 9.236 | 11.070 | 12.833 | 15.086 | 16.750 |
| 6 | 0.676 | 0.872 | 1.237 | 1.635 | 2.204 | 10.645 | 12.592 | 14.449 | 16.812 | 18.548 |
| 7 | 0.989 | 1.239 | 1.690 | 2.167 | 2.833 | 12.017 | 14.067 | 16.013 | 18.475 | 20.278 |
| 8 | 1.344 | 1.646 | 2.180 | 2.733 | 3.490 | 13.362 | 15.507 | 17.535 | 20.090 | 21.955 |
| 9 | 1.735 | 2.088 | 2.700 | 3.325 | 4.168 | 14.684 | 16.919 | 19.023 | 21.666 | 23.589 |
| 10 | 2.156 | 2.558 | 3.247 | 3.940 | 4.865 | 15.987 | 18.307 | 20.483 | 23.209 | 25.188 |
| 11 | 2.603 | 3.053 | 3.816 | 4.575 | 5.578 | 17.275 | 19.675 | 21.920 | 24.725 | 26.757 |
| 12 | 3.074 | 3.571 | 4.404 | 5.226 | 6.304 | 18.549 | 21.026 | 23.337 | 26.217 | 28.300 |
| 13 | 3.565 | 4.107 | 5.009 | 5.892 | 7.042 | 19.812 | 22.362 | 24.736 | 27.688 | 29.819 |
| 14 | 4.075 | 4.660 | 5.629 | 6.571 | 7.790 | 21.064 | 23.685 | 26.119 | 29.141 | 31.319 |
| 15 | 4.601 | 5.229 | 6.262 | 7.261 | 8.547 | 22.307 | 24.996 | 27.488 | 30.578 | 32.801 |
| 16 | 5.142 | 5.812 | 6.908 | 7.962 | 9.312 | 23.542 | 26.296 | 28.845 | 32.000 | 34.267 |
| 17 | 5.697 | 6.408 | 7.564 | 8.672 | 10.085 | 24.769 | 27.587 | 30.191 | 33.409 | 35.718 |
| 18 | 6.265 | 7.015 | 8.231 | 9.390 | 10.865 | 25.989 | 28.869 | 31.526 | 34.805 | 37.156 |
| 19 | 6.844 | 7.633 | 8.907 | 10.117 | 11.651 | 27.204 | 30.144 | 32.852 | 36.191 | 38.582 |
| 20 | 7.434 | 8.260 | 9.591 | 10.851 | 12.443 | 28.412 | 31.410 | 34.170 | 37.566 | 39.997 |
| 21 | 8.034 | 8.897 | 10.283 | 11.591 | 13.240 | 29.615 | 32.671 | 35.479 | 38.932 | 41.401 |
| 22 | 8.643 | 9.542 | 10.982 | 12.338 | 14.041 | 30.813 | 33.924 | 36.781 | 40.289 | 42.796 |
| 23 | 9.260 | 10.196 | 11.689 | 13.091 | 14.848 | 32.007 | 35.172 | 38.076 | 41.638 | 44.181 |
| 24 | 9.886 | 10.856 | 12.401 | 13.848 | 15.659 | 33.196 | 36.415 | 39.364 | 42.980 | 45.559 |
| 25 | 10.520 | 11.524 | 13.120 | 14.611 | 16.473 | 34.382 | 37.652 | 40.646 | 44.314 | 46.928 |
| 26 | 11.160 | 12.198 | 13.844 | 15.379 | 17.292 | 35.563 | 38.885 | 41.923 | 45.642 | 48.290 |
| 27 | 11.808 | 12.879 | 14.573 | 16.151 | 18.114 | 36.741 | 40.113 | 43.195 | 46.963 | 49.645 |
| 28 | 12.461 | 13.565 | 15.308 | 16.928 | 18.939 | 37.916 | 41.337 | 44.461 | 48.278 | 50.993 |
| 29 | 13.121 | 14.256 | 16.047 | 17.708 | 19.768 | 39.087 | 42.557 | 45.722 | 49.588 | 52.336 |
| 30 | 13.787 | 14.953 | 16.791 | 18.493 | 20.599 | 40.256 | 43.773 | 46.979 | 50.892 | 53.672 |
| 40 | 20.707 | 22.164 | 24.433 | 26.509 | 29.051 | 51.805 | 55.758 | 59.342 | 63.691 | 66.766 |
| 50 | 27.991 | 29.707 | 32.357 | 34.764 | 37.689 | 63.167 | 67.505 | 71.420 | 76.154 | 79.490 |
| 60 | 35.534 | 37.485 | 40.482 | 43.188 | 46.459 | 74.397 | 79.082 | 83.298 | 88.379 | 91.952 |
| 70 | 43.275 | 45.442 | 48.758 | 51.739 | 55.329 | 85.527 | 90.531 | 95.023 | 100.425 | 104.215 |
| 80 | 51.172 | 53.540 | 57.153 | 60.391 | 64.278 | 96.578 | 101.879 | 106.629 | 112.329 | 116.321 |
| 90 | 59.196 | 61.754 | 65.647 | 69.126 | 73.291 | 107.565 | 113.145 | 118.136 | 124.116 | 128.299 |
| 100 | 67.328 | 70.065 | 74.222 | 77.929 | 82.358 | 118.498 | 124.342 | 129.561 | 135.807 | 140.169 |

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