[ENG07]

UNIVERSITY OF BOLTON

ENGINEERING, SPORTS and SCIENCES

B.ENG (HONS) MECHANICAL ENGINEERING

SEMESTER ONE EXAMINATION 2018/2019

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME5012

Date: Monday 14th January 2019 **Time: 14:00 – 16:00**

INSTRUCTIONS TO CANDIDATES:

There are SEVEN questions.

Answer ANY FOUR questions ONLY.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

Electronic calculators may be used provided that data and program storage memory is cleared prior to the examination.

CANDIDATES REQUIRE: **Formula Sheet (attached).**

Q1: An element of a new spacecraft is being designed with a composite fuselage skin that is made from multiple layers of graphite reinforced epoxy. The fibres will be placed in different directions. The element is subjected to a two-dimensional stress system as shown in figure Q1.

a) Determine via calculation:

(i) The magnitude of the principal stresses. **(5 marks)**

(ii) The angular position of the principal planes in relation to the X-axis **(3 marks)**

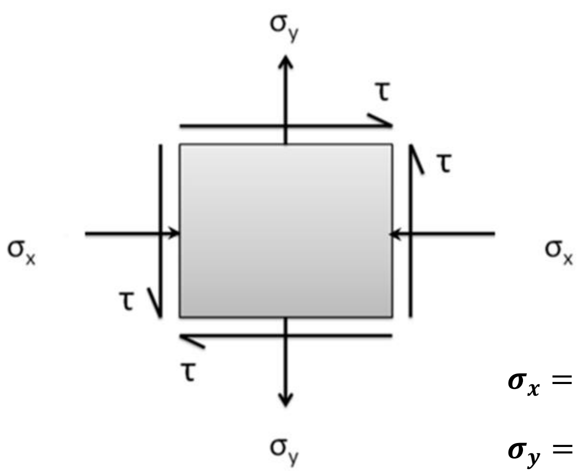
(iii) The magnitude of the maximum shear stress. **(3 marks)**

b) Sketch a Mohr's Stress Circle from the information provided in figure Q1, labelling σ_1, σ_2 the principal stresses and the maximum shear stress τ_{max}. Verify the results found in part a). **(8 marks)**

c) Illustrate on a sketch of the element:

(i) The orientation of the principal planes. **(3 marks)**

(ii) The orientation of the plane where the shear stress is maximum. **(3 marks)**



$\boldsymbol{\sigma_x = 60\ MPa}$

$\boldsymbol{\sigma_y = 50\ MPa}$

$\boldsymbol{\tau_{xy} = \tau_{yx} = 30\ MPa}$

Figure Q1

Total 25 Marks

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Q2: The purpose is to design a cantilever beam carrying an UDL ω=60kN/m as shown in figure Q2. For that purpose, some information are needed by answering the following questions applying a factor of safety of 3.

Given: E=250GPa, L=2m.

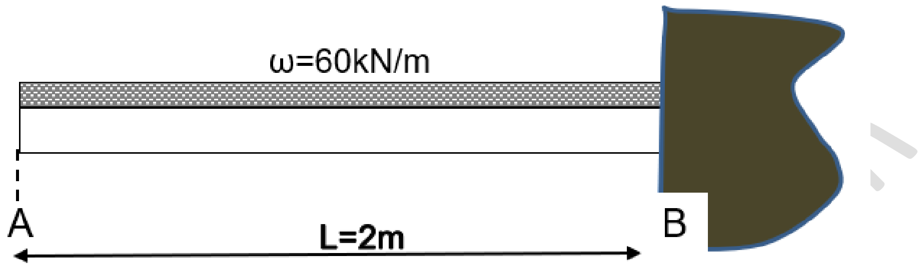

Figure Q2: Cantilever beam with UDL

a) Give the expression of the bending moment at any position along the beam in function of x. **(2 marks)**

b) Derive the formula of the maximum deflection y_{max} at the end A of the beam. **(8 marks)**

c) Calculate the flexural rigidity (EI) of the beam if the maximum allowable deflection is not to exceed 5mm. **(4 marks)**

d) Determine the dimension of the cross-section beam if it has a rectangular cross section so that the height is twice the width. **(6 marks)**

e) Calculate the allowable bending stress. **(5 marks)**

Total 25 Marks

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Q3: A part of an elevator engine is assumed to be pin connected both its ends as shown in figure Q3. AB is a solid L2-steel rod with a yield stress σ_y = 703MPa and an

elasticity modulus E=200GPa. If the load at C has a mass of 500 kg, determine:

a) Determine the compressive force F_{AB} developed in rod AB. **(4 marks)**

b) Calculate the required minimum diameter of the solid L2-steel rod AB to the nearest mm so that it will not buckle. Use a factor of safety of 2 against buckling. **(6 marks)**

c) Calculate the critical stress and use your result to discuss the validity of Euler's formula. **(4 marks)**

d) If the diameter of the solid L2-steel rod AB is now 50 mm, determine the maximum mass C that the rod can support without buckling. Use a factor of safety of 2 against buckling. **(7 marks)**

e) Calculate the new critical stress with the diameter of 50mm and use your result to discuss the validity of Euler's formula. **(4 marks)**

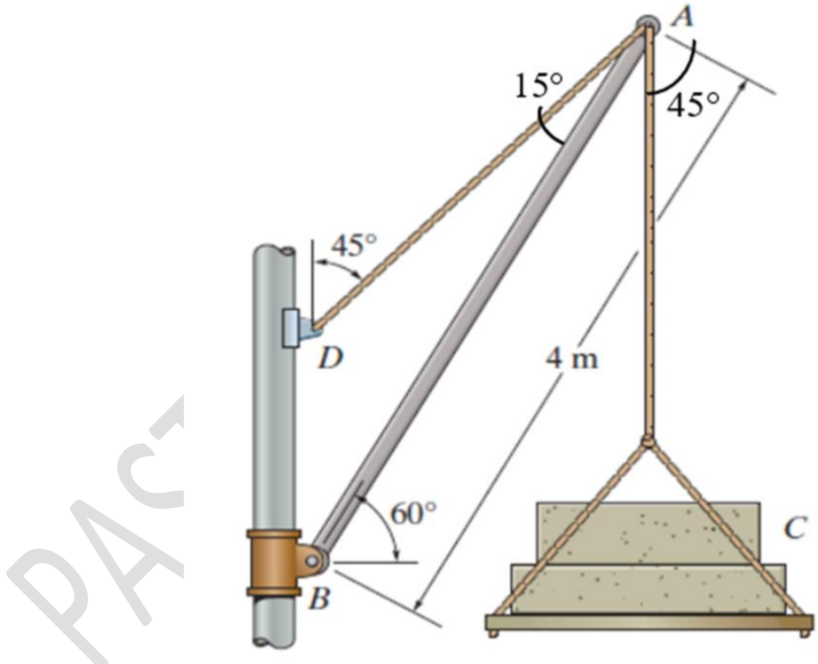

Figure Q3

Total 25 Marks

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Q4. A long, closed ended cylindrical gas pressure vessel has an outer diameter of 900mm and an inner diameter of 500mm as shown in figure Q4. If the vessel is subjected to an internal pressure until the inner circumferential layers reach 200MPa, calculate:

a) The radial stress (σ_R) at the inner and outer surfaces. Give the internal pressure. **(7 marks)**

b) The circumferential stress (σ_C) at the outer surfaces. **(2 marks)**

c) The longitudinal stress (σ_L) and the maximum shear stress **(4 marks)**

d) The circumferential strain (ε_C), the radial strain (ε_R) and the longitudinal strain (ε_L) at the inner and outer surface. **(6 marks)**

e) The final thickness, the final diameter and the final nominal volume of the cylinder. **(6 marks)**

Take E=230GPa, ν=0.3 and L=2m.

Total 25 Marks

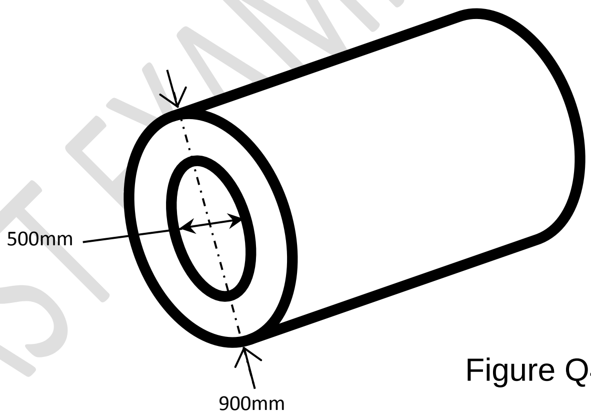

Figure Q4

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Q5. A machine of mass 1550kg is supported by four identical elastic springs coupled with a dashpot and set oscillating. It is observed that the amplitude reduces to 35% of its initial value after 5 oscillations over 10 seconds.

Calculate the following:

a) The natural frequency of undamped vibrations (in Hertz). **(2 marks)**

b) The effective stiffness of all four springs together. **(4 marks)**

c) The critical damping coefficient. **(2 marks)**

d) The damping ratio. **(5 marks)**

e) The damping coefficient. **(2 marks)**

f) The frequency of damped vibrations. **(3 marks)**

g) Explain as much as you can an underdamping, a critical and an overdamped system. **(7 marks)**

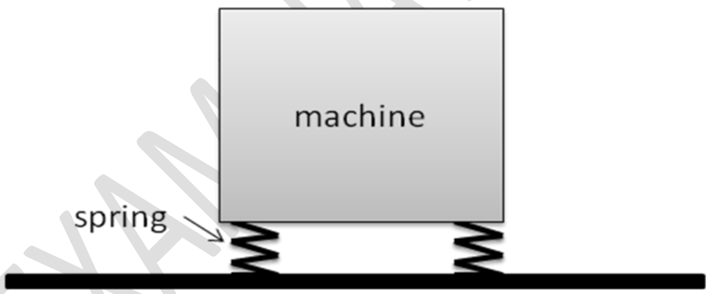

Figure Q5

Total 25 Marks

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Q6: A steel plate is to be used to fabricate a cylindrical pressure vessel with a diameter *D* of 3m and a wall thickness *t* of 20 mm. Due to the connection of a flange at the position of interest there are also shear stresses present related to xy with a value of 0.053 MPa.

a) What is the maximum internal pressure (P) allowable if the yield stress, σ_{yield}, is equal to 800 MPa and assuming a safety factor of 3?
Given the Hoop stress is Pr/t and longitudinal stress is Pr/2t.

(6 marks)

b) Draw the elemental square showing the stresses acting **(4 marks)**

c) Using this information given above calculate the principal stresses using the eigenvalues method.

(8 marks)

d) Determine the angles relative to xy co-ordinates of the largest principal stress acting and make a sketch showing the direction of the two principal stresses.

(7 Marks)

Total 25 Marks

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Q7: A 60 mm pultruded beam section for a gas rig is fabricated from glass reinforced polyester resin as shown in fig Q7. The beam has a length of 3m and assumed to be simply supported at each end. The beam needs to support a point load of 15 KN. If the beam is designed to not to exceed the maximum design strain in each material.

a) Calculate the maximum bending moment **(4 Marks)**

b) Calculate the elasticity modulus and the design stress of each material of the beam. **(10 marks)**

c) Calculate the actual stress of each material of the beam. **(6 marks)**

d) determine the number of beams needed. **(5 marks)**

Total 25 Marks

Assume for the materials used the following values;

Material	Efficiency factor (%)	Design Strain (%)	Volume fraction (%)	Reinforcement modulus (GPa)	Matrix modulus (GPa)
UD	90	0.2	68	70	3
WR	50	0.2	40	70	3

Table Q7 Material Properties

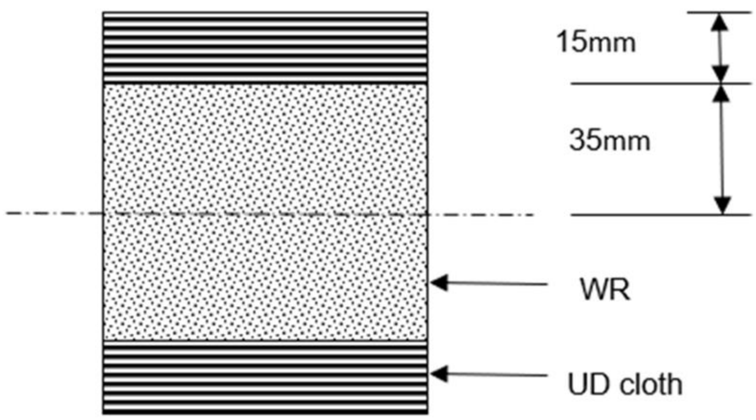

Fig. Q7: Beam Cross-Section

END OF QUESTIONS

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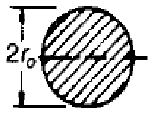
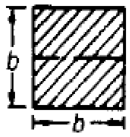
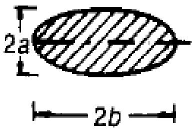
FORMULA SHEET

Deflection:

$$M_{xx} = EI\,\frac{d^2y}{dx^2}$$

Section Shape	$A(m^2)$	$I_{xx}(m^4)$
	πr^2	$\frac{\pi}{4}r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4}a^3b$

Plane Stress:

a) Stresses in function of the angle Θ:

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2}sin(2\theta) + \tau_{xy}\,cos(2\theta)$$

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b) Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

$$\tau_{\max} = \frac{1}{2}\sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} \qquad \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Lame's equation

The equations are known as "Lame's Equations" for radial and hoop stress at any specified point on the cylinder wall. Note: R_1 = inner cylinder radius, R_2 = outer cylinder radius

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = 1/E\ \{\sigma_c - \nu(\sigma_r + \sigma_L)\}$$

$$\varepsilon_r = 1/E\ \{\sigma_r - \nu(\sigma_c + \sigma_L)\}$$

$$\varepsilon_L = 1/E\ \{\sigma_L - \nu(\sigma_c + \sigma_r)\}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)} \qquad \tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

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Vibrations:

Free Vibrations:

$$f = \frac{1}{T} \qquad \omega_n = 2\pi f = \sqrt{\frac{k}{\mathrm{M}}}$$

Damped Vibrations:

$$f_d = \frac{\omega_d}{2\pi} \qquad c_c = \sqrt{4\mathrm{M}k} \qquad \zeta = \frac{c}{c_c} = \frac{c}{2k}\,\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi a\zeta}{\sqrt{1-\zeta^2}}$$, a is the number of oscillations

Stress

σ = Force/Area = F/A

Hook's law

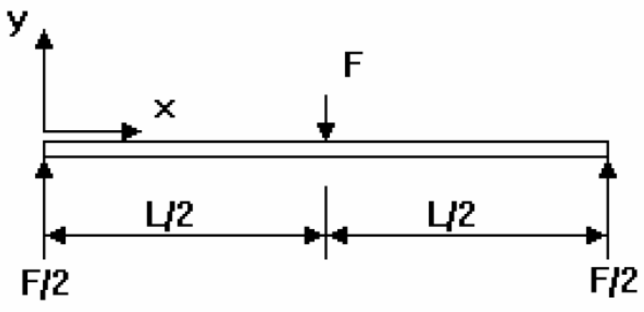
σ = E·ε

ε = ΔL/L

Composite Materials

Rule of mixture: $E_c = \eta V_F E_F + V_m E_m$
$\eta = efficiency\ factor$
$V_F = volume\ fraction\ of\ fibre$
$E_F = elasticity\ modulus\ of\ fibre$
$V_m = volume\ fraction\ of\ resin\ matrix$
$E_m = elasticity\ modulus\ of\ resin\ matrix$

Simply supported beam:



M: maximum bending moment (M_{max}=FL/4)

Maximum bending stress:

$$\sigma_{bending} = \frac{My}{I}$$

M: maximum bending moment
Y: distance from neutral axis
I: second moment of area

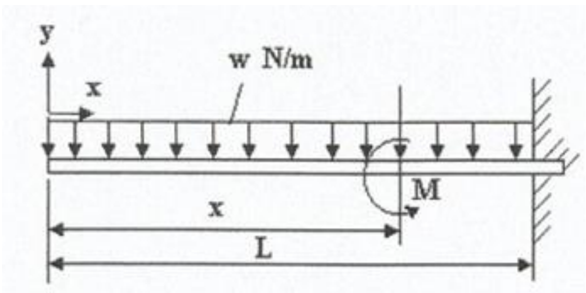
Slope at the ends:

$$\frac{dy}{dx} = \frac{FL^2}{16EI}$$

Maximum deflection at the middle:

$$y = \frac{FL^3}{48EI}$$

Cantilever with uniformed distributed load (UDL)



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M: maximum bending moment ($M_{max}=wL^2/2$)

Maximum bending stress:

$$\sigma_{bending} = \frac{My}{I}$$

M: maximum bending moment
Y: distance from neutral axis
I: second moment of area

Slope at the ends:

$$\frac{dy}{dx} = \frac{wL^3}{6EI}$$

Maximum deflection at the middle:

$$y = \frac{wL^4}{8EI}$$

Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = (Stress\ Tensor)\begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2+b^2+c^2}}$$

Where a, b and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \qquad l = \cos\alpha,$$
$$m = bk \qquad m = \cos\theta,$$
$$n = ck \qquad n = \cos\varphi.$$

Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

Yield Criterion

Von Mises

$$\sigma_{von\ Mises} = \frac{1}{\sqrt{2}}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]^{1/2}$$

Tresca

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

$$\sigma_{tresca} = 2 \cdot \tau_{\max}$$

$$\tau_{\max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}; \frac{|\sigma_1 - \sigma_3|}{2}; \frac{|\sigma_3 - \sigma_2|}{2}\right)$$

$$\frac{\sigma_{von\ Mises}}{\sigma_{Tresca}} = \frac{\sqrt{3}}{2}$$

Quadratic equation: $ax^2+bx+c=0$

Solution:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Allowable stress: $\boldsymbol{\sigma_{allowable}}$

$$\boldsymbol{\sigma_{allowable}} = \frac{\boldsymbol{\sigma_{yield}}}{\boldsymbol{Factor\ Of\ Safety}}$$

Struts:

$$I = k^2 A$$

$$k = \sqrt{\frac{I}{A}}$$

Euler validity

$$Slenderness\ ratio = SR = \frac{L_e}{k} \geq \pi \sqrt{\frac{E}{\sigma_{yield}}}$$

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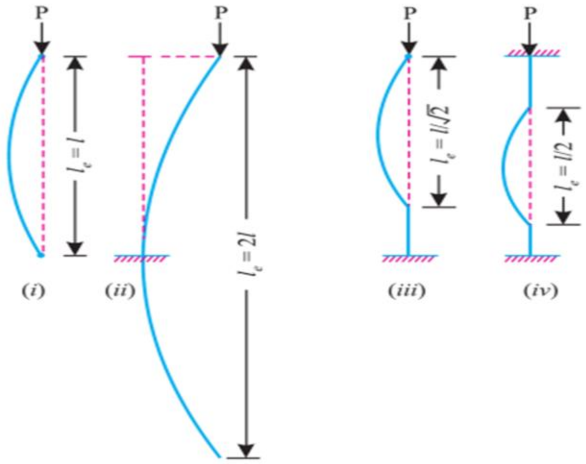

(i) *Both ends pin jointed or hinged or rounded or free.*
(ii) *One end fixed and other end free.*
(iii) *One end fixed and the other pin jointed.*
(iv) *Both ends fixed.*

Case	*End conditions*	*Equivalent length,* l_e	*Buckling load, Euler*
1	*Both ends hinged or pin jointed* or *rounded or free*	l	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{l^2}$
2.	*One end fixed, other end free*	$2l$	$\frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{4l^2}$
3.	*One end fixed, other end pin jointed*	$\frac{l}{\sqrt{2}}$	$\frac{\pi^2 EI}{l_e^2} = \frac{2\pi^2 EI}{l^2}$
4.	*Both ends fixed or encastered*	$\frac{l}{2}$	$\frac{\pi^2 EI}{l_e^2} = \frac{4\pi^2 EI}{l^2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c \cdot A}{1 + a \cdot \left(\frac{l_e}{k}\right)^2}$$

We find,

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a\left(\frac{l_e}{k}\right)^2$ has thus been introduced to *take into account the buckling effect.*

$$a = \frac{\sigma_c}{\pi^2 \cdot E}$$

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