UNIVERSITY OF BOLTON

OFF-CAMPUS DIVISION

B.ENG. (HONS) MECHANICAL ENGINEERING

MALAYSIA - KTG

SEMESTER 1 EXAMINATION 2018/2019

MECHANICS OF MATERIALS AND MACHINES

MODULE NO: AME 5002

Date: Wednesday 9th January 2019 Time: 2 Hours

INSTRUCTIONS TO CANDIDATES:

There are FOUR questions.

Answer ALL questions.

All questions carry equal marks.

Marks for parts of questions are shown in brackets.

This examination paper carries a total of 100 marks.

All working must be shown. A numerical solution to a question obtained by programming an electronic calculator will not be accepted.

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Q1. The plane element in Figure Q1 is subjected to the stresses shown.

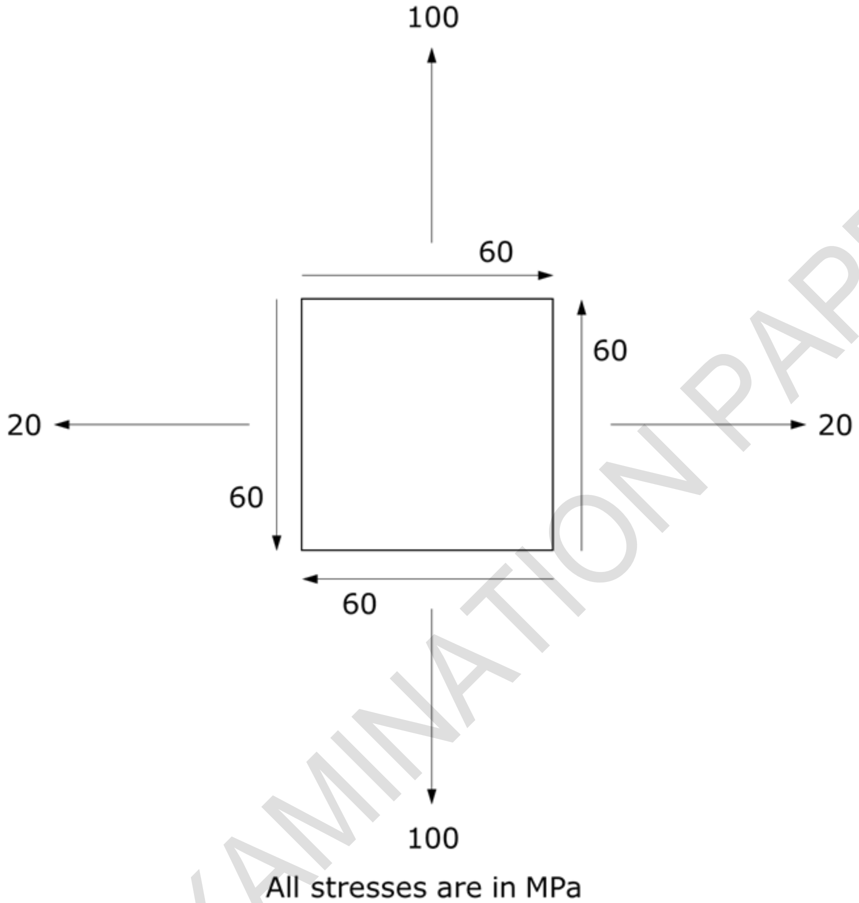

Figure Q1

(a) Create a drawing of the Mohr's Circle for the element. (7 marks)

(b) Evaluate the principle stresses. (5 marks)

(c) Solve the maximum possible shear stress. (5 marks)

(d) Solve the angle of the plane of maximum positive shear stress with respect to the x-plane. Generate a sketch of the element with the orientation of this plane. (8 marks)

Total 25 marks
Please turn the page

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Q2. The beam ABCD as shown in Figure Q2 has a span of 14 m and is simply-supported at both ends.

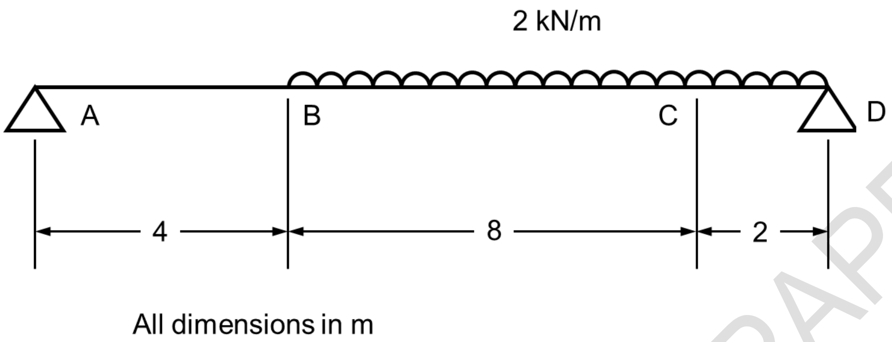

Figure Q2

Use Macaulay's method to evaluate the deflection at the centre of the span (position C).
[The flexural rigidity of the beam EI is 8×10^4]. (25 marks)

Total 25 marks

Q3. There is a 5 m long steel strut of circular cross-section with the outer diameter of 80 mm and inside diameter of 70 mm. One of the steel strut is subjected to the line of action of 60 kN of the thrust parallel to the unstrained line of the strut, but not coincide with it. With the load, the steel strut possesses the maximum deflection expressed as:

$$y_{max} = e\left[\sec\left(\frac{\alpha L}{2}\right) - 1\right]$$

where e is the eccentricity of the compressive which is 25 mm, axial load which acts parallel to the axis of the strut, and $\alpha = \sqrt{P/EI}$ where P is the applied load, E is Young's modulus of the material, and I is the second moment of area. The ends are assumed to be hinged. Assume $E = 209$ GPa for steel. Solve the maximum compressive stress of the strut. (25 marks)

Total 25 marks

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Q4. The 150 N piston is supported by a spring of modulus $k = 300$ N/m. A dashpot of damping coefficient $c = 90$ Ns/m acts in parallel with the spring. A fluctuating pressure $p = 0.8 \sin 30t$ in N/m^2 acts on the piston, whose top surface area is 95 m^2. Evaluate the steady-state displacement as a function of time and the maximum force transmitted to the base.

(25 marks)

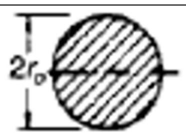
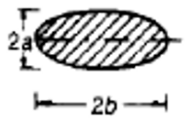
Total 25 marks

END OF QUESTIONS

Formula Sheet

1. Deflection

$$M_{xx} = EI\frac{d^2y}{dx^2}$$

Section shape	A (m^2)	I_{xx} (m^4)
	πr^2	$\frac{\pi}{4}r^4$
	b^2	$\frac{b^4}{12}$
	πab	$\frac{\pi}{4}a^3b$

2. Plane stress

Stresses in function of the angle θ:

$$\sigma_x(\theta) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y(\theta) = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}(\theta) = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta)$$

Principal stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y^2\right)^2 + 4\tau_{xy}^2}$$

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$$\tau_{max} = \frac{1}{2}\sqrt{\left(\sigma_x - \sigma_y^2\right)^2 + 4\tau_{xy}^2}$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

3. Lame's equation

$$\sigma_C = a + \frac{b}{r^2}$$

$$\sigma_R = a - \frac{b}{r^2}$$

$$\sigma_L = \frac{P_1 R_1^2 - P_2 R_2^2}{(R_2^2 - R_1^2)}$$

$$\tau_{max} = \frac{\sigma_c - \sigma_r}{2} = \frac{b}{r^2}$$

The corresponding strains format is:

$$\varepsilon_c = \frac{1}{E}[\sigma_c - \nu(\sigma_r + \sigma_l)]$$

$$\varepsilon_r = \frac{1}{E}[\sigma_r - \nu(\sigma_c + \sigma_l)]$$

$$\varepsilon_l = \frac{1}{E}[\sigma_l - \nu(\sigma_c + \sigma_r)]$$

4. Vibrations

Free vibrations:

$$f = \frac{1}{T} \qquad \omega_n = 2\pi f = \sqrt{\frac{k}{M}}$$

Damped vibration:

$$f_d = \frac{\omega_d}{2\pi} \qquad c_c = \sqrt{4Mk} \qquad \delta = \frac{c}{c_c} = \frac{c}{2k}\omega_n$$

$$\omega_d = \omega_n\sqrt{1-\delta^2}$$

$$\ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\delta}{\sqrt{1-\delta^2}}$$

$$x = x_0\cos\omega_n t + \frac{\dot{x}_0}{\omega_n}\sin\omega_n t$$

$$x = \sqrt{x_0^2 + \left(\frac{\dot{x}_0}{\omega_n}\right)^2}\sin\left[\omega_n t + \tan^{-1}\left(\frac{x_0\omega_n}{\dot{x}_0}\right)\right]$$

$$X = \frac{F_0/k}{\{[1-(\omega/\omega_n)^2]^2 + [2\zeta\,\omega/\omega_n]^2\}}$$

$$\phi = \tan^{-1}\left[\frac{2\zeta\,\omega/\omega_n}{1-(\omega/\omega_n)^2}\right]$$

$$x_p = X\sin(\omega t - \phi)$$

$$F_{tr} = kx_p + c\dot{x}_p$$

$$F_{tr,max} = \sqrt{(kX)^2 + (c\omega X)^2}$$

5. Differential equation

Homogeneous form:

$$a\ddot{y} + b\dot{y} + cy = 0$$

Characteristic equation:

$$a\lambda^2 + b\lambda + c = 0$$

If $b^2 - 4ac > 0$, λ_1 and λ_2 are distinct real numbers then the general solution of the differential equation is:

$$y(t) = Ae^{\lambda_1 t} + Be^{\lambda_2 t}$$

A and B are constant.

If $b^2 - 4ac = 0$, $\lambda_1 = \lambda_2 = \lambda$ are distinct real numbers then the general solution of the differential equation is:

$$y(t) = e^{\lambda t}(A + Bx)$$

A and B are constant.

If $b^2 - 4ac < 0$, λ_1 and λ_2 are complex numbers then the general solution of the differential equation is:

$$y(t) = e^{\alpha t}[A\cos(\beta t) + B\sin(\beta t)]$$
$$\alpha = -\frac{b}{2a}$$
$$\beta = \frac{\sqrt{4ac - b^2}}{2a}$$

A and B are constant.

6. Asymmetrical bending

$$I_{u,v} = \frac{1}{2}\left(I_{xx} + I_{yy}\right) \pm \frac{1}{2}\left(I_{xx} - I_{yy}\right)\sec 2\theta$$
$$\tan 2\theta = \frac{2I_{xy}}{I_{yy} - I_{xx}}$$
$$I_{xy} = Ahk$$
$$I_u + I_v = I_{xx} + I_{yy}$$
$$\sigma = \frac{M_v U}{I_v} + \frac{M_u V}{I_u}$$
$$\sigma_{bending} = \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

7. Stress

$$\sigma = \frac{F}{A}$$

8. Hooke's law

$$E = \frac{\sigma}{\varepsilon}$$

$$\varepsilon = \frac{\Delta L}{L}$$

9. Beam bending equation

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

10. Elasticity – finding the direction vectors

$$\begin{bmatrix} S_x \\ S_y \\ S_z \end{bmatrix} = \text{(Stress tensor)} \begin{pmatrix} l \\ m \\ n \end{pmatrix}$$

$$k = \frac{1}{\sqrt{a^2 + b^2 + c^2}}$$

where a, b, and c are the co-factors of the eigenvalue stress tensor.

$$l = ak \qquad l = \cos\alpha$$

$$m = bk \qquad m = \cos\theta$$

$$n = ck \qquad n = \cos\varphi$$

11. Principal stresses and Mohr's Circle

$$\tau_{12} = \frac{\sigma_1 - \sigma_2}{2}$$

$$\tau_{13} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\tau_{23} = \frac{\sigma_2 - \sigma_3}{2}$$

12. Yield criterion

Von Mises:

$$\sigma_{vm} = \frac{1}{\sqrt{2}}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Tresca:

$$\sigma_3 \geq \sigma_2 \geq \sigma_1$$

$$\sigma_{tr} = 2\tau_{max}$$

$$\tau_{max} = \max\left(\frac{|\sigma_1 - \sigma_2|}{2}; \frac{|\sigma_1 - \sigma_3|}{2}; \frac{|\sigma_3 - \sigma_2|}{2}\right)$$

$$\frac{\sigma_{vm}}{\sigma_{tr}} = \frac{\sqrt{3}}{2}$$

13. Quadratic equation: $\boldsymbol{ax^2 + bx + c = 0}$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

14. Allowable stress

$$\sigma_{allowable} = \frac{\sigma_{yield}}{\text{Factor of safety}}$$

15. Strut

$$k = \sqrt{\frac{I}{A}}$$

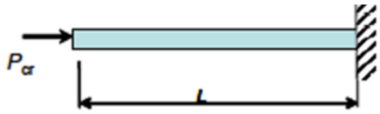
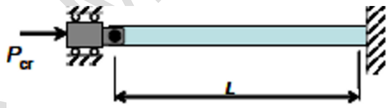
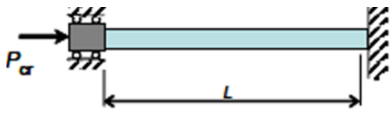
Euler validity:

$$\sigma_E = \frac{n\pi^2 E}{(L/k)^2}$$

Rankine-Gordon:

$$\sigma_R = \frac{\sigma}{1 + c/n\,(L/k)^2}$$

Slenderness ratio $= SR = \frac{L_e}{k} \geq \pi\sqrt{\frac{E}{\sigma_{yield}}}$

Description	**Schematic**	**Critical buckling load P_c**	**Effective length L_{eff}**
Free-fixed		$P_c = \frac{\pi^2 EI}{4l^2}$	$2l$
Hinged-hinged		$P_c = \frac{\pi^2 EI}{l^2}$	l
Hinged-hinged, initially curved		$P_c = \frac{\pi^2 EI}{l^2}$	l
Fixed-hinged		$P_c = \frac{2.045\pi^2 EI}{l^2}$	$0.7l$
Fixed-fixed		$P_c = \frac{4\pi^2 EI}{l^2}$	$\frac{l}{2}$

Studying Rankine's formula,

$$P_{Rankine} = \frac{\sigma_c A}{1 + a\left(\frac{l_e}{k}\right)^2}$$

We find

$$P_{Rankine} = \frac{\text{Crushing load}}{1 + a\left(\frac{l_e}{k}\right)^2}$$

The factor $1 + a(l_e/k)^2$ has thus been introduced to *take into account the buckling effect*.

$$a = \frac{\sigma_c}{\pi^2 E}$$

16. Composite materials

$$\sigma = \frac{My}{I}$$
$$E = \eta V_f E_f + \left(1 - V_f\right)E_m$$
$$\sigma = E\varepsilon$$