# **UNIVERSITY OF BOLTON**

# **INSTITUTE OF MANAGEMENT**

# **BA (HONS) ACCOUNTANCY**

# **SEMESTER 1 2018/2019**

# **QUANTITATIVE METHODS FOR ACCOUNTANTS**

**MODULE NO: ACC4018** 

Date: Thursday 17<sup>th</sup> January 2019 Time: 10.00 – 1.00

### **INSTRUCTIONS TO CANDIDATES:**

There are four compulsory questions on this paper.

Answer all four questions.

All questions carry equal marks.

Calculators may be used but full workings must be shown.

Formulae books, which contain statistical tables.

Graph paper (four sheets).

#### **Question 1**

A factory produces two products: Saffron and Silk. The contribution to profit that can be obtained is £25 per unit from Saffron, and £35 per unit from Silk. The factory employs 200 skilled workers and 150 unskilled workers, and they work a 40 hour week. The time required to produce 1 unit of Saffron is 6 skilled hours and 4 unskilled hours, whilst for 1 unit of Silk is 5 skilled hours and 7 unskilled hours.

a) Arrange the given information into tabular form.

(2 Marks)

b) Translate the problem into a linear programming one, identifying and writing down the objective function and the constraints.

(3 marks)

c) Plot the inequalities on a graph and identify the feasible region.

(10 marks)

d) Find the optimum solution that satisfies the objective function.

(10 marks)

(Total 25 marks)

Please turn the page

#### Question 2

A college girl takes part in a shot put competition. She has three attempts at throwing the shot put and scoring the highest score.

The probabilities are as follows:

She has a 0.7 probability of successfully scoring the highest score at her first attempt.

If she succeeds at the first attempt, the same probability applies on the next two attempts.

If she is not successful at any time, the probability of succeeding on any subsequent attempts is only 0.2.

Use a tree diagram to find the probabilities that:

a) Draw a tree diagram to show the probabilities of success or failure

(5 marks)
b) She is successful on all her first three attempts.

(5 marks)
c) She fails at the first attempt but succeeds on the next two.

(5 marks)

e) She is still not successful after the third attempt

d) She is successful just once in three attempts

(5 marks)

(5 marks)

(Total 25 marks)

Please turn the page

#### **Question 3**

The Table below shows a sample of 40 patients age on a hospital ward.

26	52	37	61	20	59	47	31
35	28	53	34	62	31	52	44
57	40	21	55	45	49	25	26
18	65	44	51	39	39	41	51
31	39	55	38	43	37	60	34

a) Produce a grouped frequency distribution (GFD) table for this data. (5 marks)

b) Draw a histogram of the grouped frequency distribution, and on the same graph estimate the mode age.

(5 marks)

c) From the GFD calculate the mean deviation

(5 marks)

d) From the GFD calculate the mean age.

(5 marks)

e) Calculate the corresponding variance and standard deviation.

(5 marks)

(Total 25 marks)

#### **Question 4**

A factory producing hand carved wooden tables made from indian rosewood wants to determine the relationship between the cost of output and the number of tables (units) produced.

The cost of output is thought to depend on the number of units produced.

The table below shows a record for a random sample over 10 months. Data shows:

Month	Output (Units)	Cost (£'000)			
		Y			
1	4	5			
2	6	7			
3	2	4			
4	8	9			
5	6	7			
6	10	14			
7	5	6			
8	1	2			
9	3	3			
10	5	6			

#### Required:

Please show all calculation workings.

a) Draw a scatter diagram of these results.

(5 marks)

b) Calculate the equation of the least square regression line of "y on x" and then draw this line on the scatter diagram.

(10marks)

c) Calculate the Pearson's correlation coefficient, r and the coefficient of determination r<sup>2</sup>.

(6 Marks)

d) Use the regression equation/line to predict the likely cost of 2 months if output is 7, and 9 respectively.

(4 marks) (Total 25 marks)

**END OF QUESTIONS** 

### STATISTICAL FORMULAE

## FREQUENCY DISTRIBUTIONS

Required fractile from a GFD = Lower class limit of fractile class +

Fractile item - cumulative frequency Fractile up to lower class limit of fractile class × class Fractile class frequency interval

Mean  $\bar{x} = \frac{\text{sum of values}}{\text{total number of items}} = \frac{\sum x}{n}$ 

with GFD:  $\bar{x} = \frac{\sum (f \times MP)}{\sum f}$  MP = class Mid Point

Range = Highest value - Lowest value

Quartile deviation =  $(Q_3 - Q_1)/2$ 

Mean deviation =  $\frac{\sum (x - \overline{x})}{n}$  The sign of  $(x - \overline{x})$  must be ignored

with GFD: M.D. =  $\frac{\sum (f \times (MP - \overline{x}))}{\sum f}$ 

Standard deviation (s) =  $\sqrt{\frac{\sum (x - \overline{x})^2}{n}}$ 

If the mean is not a rounded number:  $\mathbf{s} = \sqrt{\frac{\sum x^2}{n} - \overline{x}^2}$ 

with GFD:  $s = \sqrt{\frac{\sum (f \times MP^2)}{\sum f} - \overline{x}^2}$ 

Variance: s<sup>2</sup>

Coefficient of variation =  $\frac{s}{\overline{z}} \times 100$ 

3 (Mean – Median) Pearson's Coefficient of Skewness (Sk) = Standard Deviation

### CORRELATION

Regression line of "y on x": y = a + bx

be of my off 
$$\mathbf{x}$$
.  $\mathbf{y} = \mathbf{a} + b\mathbf{x}$ 

$$\mathbf{b} = \frac{\mathbf{n} \times \sum \mathbf{xy} - \sum \mathbf{x} \times \sum \mathbf{y}}{\mathbf{n} \times \sum \mathbf{x}^2 - (\sum \mathbf{x})^2} \qquad \mathbf{a} = \frac{\sum \mathbf{y} - \mathbf{b} \times \sum \mathbf{x}}{\mathbf{n}} \qquad \mathbf{n} = \text{number of pairs}$$

$$\mathbf{a} = \frac{\sum y - b \times \sum x}{n}$$

Regression line of "x on y": x = a + by

$$\mathbf{b} = \frac{\mathbf{n} \times \sum \mathbf{y} \mathbf{x} - \sum \mathbf{y} \times \sum \mathbf{x}}{\mathbf{n} \times \sum \mathbf{y}^2 - (\sum \mathbf{y})^2} \qquad \mathbf{a} = \frac{\sum \mathbf{x} - \mathbf{b} \times \sum \mathbf{y}}{\mathbf{n}}$$

$$\mathbf{a} = \frac{\sum \mathbf{x} - \mathbf{b} \times \sum \mathbf{y}}{\mathbf{n}}$$

Pearson product-moment Coefficient of Correlation (r)

$$\mathbf{r} = \frac{\mathbf{n} \times \sum \mathbf{x} \mathbf{y} - \sum \mathbf{x} \times \sum \mathbf{y}}{\sqrt{\left(\left(\mathbf{n} \times \sum \mathbf{x}^2 - \left(\sum \mathbf{x}\right)^2\right)\left(\mathbf{n} \times \sum \mathbf{y}^2 - \left(\sum \mathbf{y}\right)^2\right)\right)}}$$

$$\mathbf{r}^2 = \mathbf{b}_{yx} \times \mathbf{b}_{xy}$$
  $\Rightarrow$   $\mathbf{r} = \sqrt{\mathbf{b}_{yx} \times \mathbf{b}_{xy}}$ 

$$\mathbf{r} = \sqrt{b_{yx} \times b_{xy}}$$

Covariance: Cov (x,y) = 
$$\frac{\sum (x - \overline{x})(y - \overline{y})}{n}$$

$$\Rightarrow \frac{\mathbf{r} = \operatorname{Cov}(\mathbf{x}, \mathbf{y})}{(\mathbf{s}_{\mathbf{x}} \times \mathbf{s}_{\mathbf{y}})}$$

Spearman's Coefficient of Rank Correlation:

$$\mathbf{r'} = 1 - \frac{6\sum d^2}{n(n^2 - 1)}$$

where

d = the difference between the rankings of the same item in each series

## **PROBABILITY**

Multiplication rule: the prob. of a sequential event is the product of all its elementary events  $P(A \cap B \cap C \cap ...) = P(A) \times P(B) \times P(C) ...$ 

Addition rule: the prob. of one of a number of mutually exclusive events occurring is the sum of the  $P(X \cup Y \cup Z \cup ...) = P(X) + P(Y) + P(Z) ...$ probabilities of the events

Bayes' Theorem 
$$P(E \mid S) = \frac{P(E) \times P(S \mid E)}{\sum_{i} (P(E_{i}) \times P(S \mid E_{i}))}$$

where

S is the subsequent event and there are n prior events, E.

#### PROBABILITY DISTRIBUTIONS

Binomial distribution

$$P(x) = \binom{n}{x} p^x q^{n-x}$$

 $P(x) = {n \choose x} p^x q^{n-x}$  where p = constant probability of a success q = 1 - p = probability of a failure

$$Mean = np$$

Standard deviation =  $\sqrt{npq}$ 

Poisson distribution

$$q = 1 - p = \text{probability}$$

$$\text{Mean} = np$$

$$\text{Standard deviation} = \sqrt{}$$

$$P(x) = e^{-a} \frac{a^{x}}{x!}$$

$$\text{where } e \cong 2.718 \text{ is a constant}$$

$$\text{Mean} = a = np$$

Mean = 
$$a = np$$
  
Standard deviation =  $\sqrt{a}$ 

Simplified Poisson

$$P(x+1) = P(x) \times \frac{a}{x+1}$$

Normal distribution: standardised value  $z = \frac{x - \mu}{\sigma}$ 

where  $\mu$  and  $\sigma$  are the mean and standard deviation of the actual distribution

### **ESTIMATION & CONFIDENCE INTERVALS**

- $\bar{x}$ , s, p sample mean, standard deviation, proportion/percentage
- 0  $\mu$ ,  $\sigma$ ,  $\pi$  – population mean, standard deviation, proportion/percentage
- $\overline{x}$  is a point estimate of  $\mu$ s is a point estimate of  $\sigma$ p is a point estimate of  $\pi$

Confidence intervals for a population percentage or proportion

$$\pi = p \pm z \sqrt{\frac{p(100 - p)}{n}}$$
 for a percentage

 $\pi = p \pm z \sqrt{\frac{p(1-p)}{n}}$  for a proportion

When using normal tables:  $\alpha = 100 - \text{confidence level}$ 

Estimation of population mean ( $\mu$ ) when  $\sigma$  is known

$$\mu = \overline{x} \pm \mathbf{z} \, \sigma / \sqrt{n}$$

(normal tables for z)

Estimation of population mean ( $\mu$ ) for large sample size and  $\sigma$  unknown

$$\mu = \bar{x} \pm z \, s / \sqrt{n}$$

(normal tables for z)

Estimation of population mean ( $\mu$ ) for small sample size and  $\sigma$  unknown

$$\mu = \bar{x} \pm t \, s / \sqrt{n}$$

(t-tables for t)

When using t-tables: v = n-1

Confidence intervals for paired (dependent) data

$$\mu_{\rm d} = \overline{x_{\rm d}} \pm t \, s_{\rm d} / \sqrt{n_{\rm d}}$$

where "d" refers to the calculated differences

### FINANCIAL MATHEMATICS

Simple interest 
$$A_n = P\left(1 + \frac{i}{100} \times n\right)$$

Compound interest 
$$A_m = P\left(1 + \frac{i}{100}\right)^m$$

Effective APR = 
$$\left(\left(1 + \frac{i}{100}\right)^{n} - 1\right) \times 100\%$$

Straight line depreciation 
$$A_n = P\left(1 - \frac{i}{100} \times n\right)$$

Depreciation 
$$A = P\left(1 - \frac{i}{100}\right)^m$$

The future value of an initial investment  $A_0$  is given by  $A = A_0 \left(1 + \frac{i}{100}\right)^n$  and the present value of an accumulated investment  $A_n$  is given by  $A_0 = \frac{A_n}{\left(1 + \frac{i}{100}\right)^n}$  or  $A\left(1 + \frac{i}{100}\right)^{-n}$ 

#### Loan account

If an annuity is purchased for a sum of  $A_0$  at a rate of i% compounded each period then the periodic repayment is

$$R = \frac{iA_0}{1 - (1 + i)^{-n}}$$

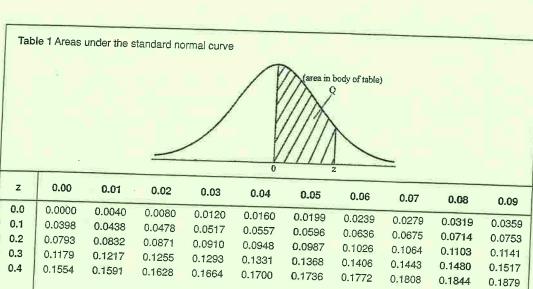
and the present value of the annuity  $A_0$  (the loan) is

$$A_0 = \mathbb{R} \times \frac{(1+i)^n - 1}{i(1+i)^n} \text{ or equivalently } A_0 = \frac{\mathbb{R}[1 - (1+i)^{-n}]}{i}$$

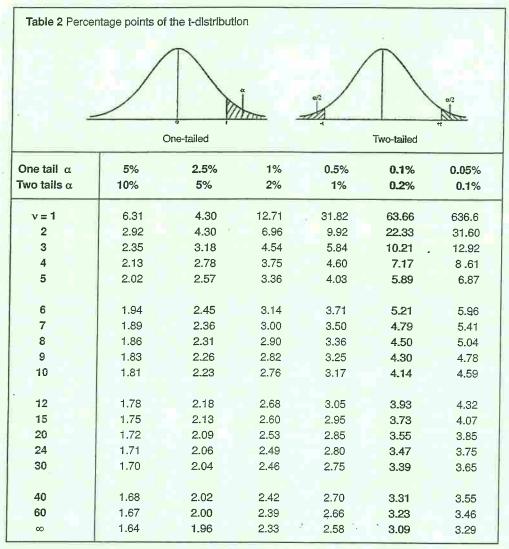
## Savings account

A savings plan/sinking fund invested for n periods at a nominal rate of i% compounded each period with a periodic investment of LP matures to I where

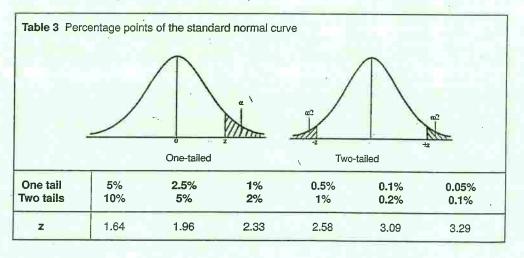
$$S = P(1+i) \times \left(\frac{1+i)^{n}-1}{i}\right)$$



			0.00	0.07	0.02	0.03	0.04	4 0.0	5 0.0	6 0.07	7 0.08	3 0.0	29
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	0.	- 1	0.1179	0.1217	0.125	5 0.129							
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	0.7	1	0.2580	0.2611	0.2642	0.2673							
-	0.8	- 1	0.2881	0.2910	0.2939	0.2967	0.2995						
1	0.9	) (	0.3159	0.3186	0.3212	0.3238							
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	1.1	_	.3643	0.3665	0.3686	0.3708	0.3729	0.3749					
1	1.2	100	.3849	0.3869	0.3888	0.3907	0.3925	0.3944					
1	1.3		.4032	0.4049	0.4066	0.4082	0.4099	0.4115					
1	1.4	10	.4192	0.4207	0.4222	0.4236	0.4251	0.4265			0.4306	0.417	- 4
1	4 5		1200							01.202	0.4000	0.4319	9
l	1.5	- 1	4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441	.
1	1.6 1.7	- 1	4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515		0.4535	0.4545	
	1.8	- 1	4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633	
	1.9	1	4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706	- 11
	1.9	0.4	4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750		0.4761	0.4767	- 1
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	2.1		1821		0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817	1
	2.2	1	1861	0.4826 0.4864	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857	
	2.3	4	893	0.4896	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890	1
	2.4			0.4920	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916	1
			0.0	0.4320	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936	1
	2.5	0.4	938	0.4940	0.4941	0.4040	0.40.5						1
	2.6			0.4955	0.4956	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952	1
	2.7	0.49		0.4966	0.4967	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964	1
	2.8	0.49		0.4975	0.4976	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974	
2	2.9	0.49		0.4982	0.4982	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981	1
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3	.2	0.49				0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993	
	.3	0.49				0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995	
3.	4	0.49				a II.	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997	
-	1					0.4007	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998	



 $\nu = degrees$  of freedom  $\alpha = total$  percentage in tails



 $\alpha$  = total percentage in tails